

Lecture notes for “Analysis of Algorithms”: Dynamic All-Pairs Shortest Paths

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<hr/> Function $\text{apsp}(G = (V, E, c))$ <hr/>	<hr/> Function $\text{insert-edges}(E_{ins})$ <hr/>
$t \leftarrow 0$	foreach $e = (u, v) \in E_{ins}$ do
foreach $u \in V$ do	$\text{insert}(E[u], e)$
$E[u] \leftarrow \emptyset$	$\text{insert}(E[v], e)$
$\pi[u] \leftarrow \text{path}(u)$	$\pi[e] \leftarrow \text{path}(e)$
$d[u, u] \leftarrow 0$	$\text{heap-insert}(P[u, v], \pi[e], \text{cost}[e])$
$p[u, u] \leftarrow \pi[u]$	
foreach $u \neq v \in V$ do	<hr/> Function $\text{delete-edges}(E_{del})$ <hr/>
$P[u, v] \leftarrow \text{heap}()$	foreach $e = (u, v) \in E_{ins}$ do
$\text{insert-edges}(E)$	$\text{delete}(E[u], e)$
$\text{build-paths}()$	$\text{delete}(E[v], e)$
	$\text{remove-path}(\pi[e])$

References

- [DI04] C. Demetrescu and G.F. Italiano. A new approach to dynamic all pairs shortest paths. *Journal of the ACM*, 51(6):968–992, 2004.

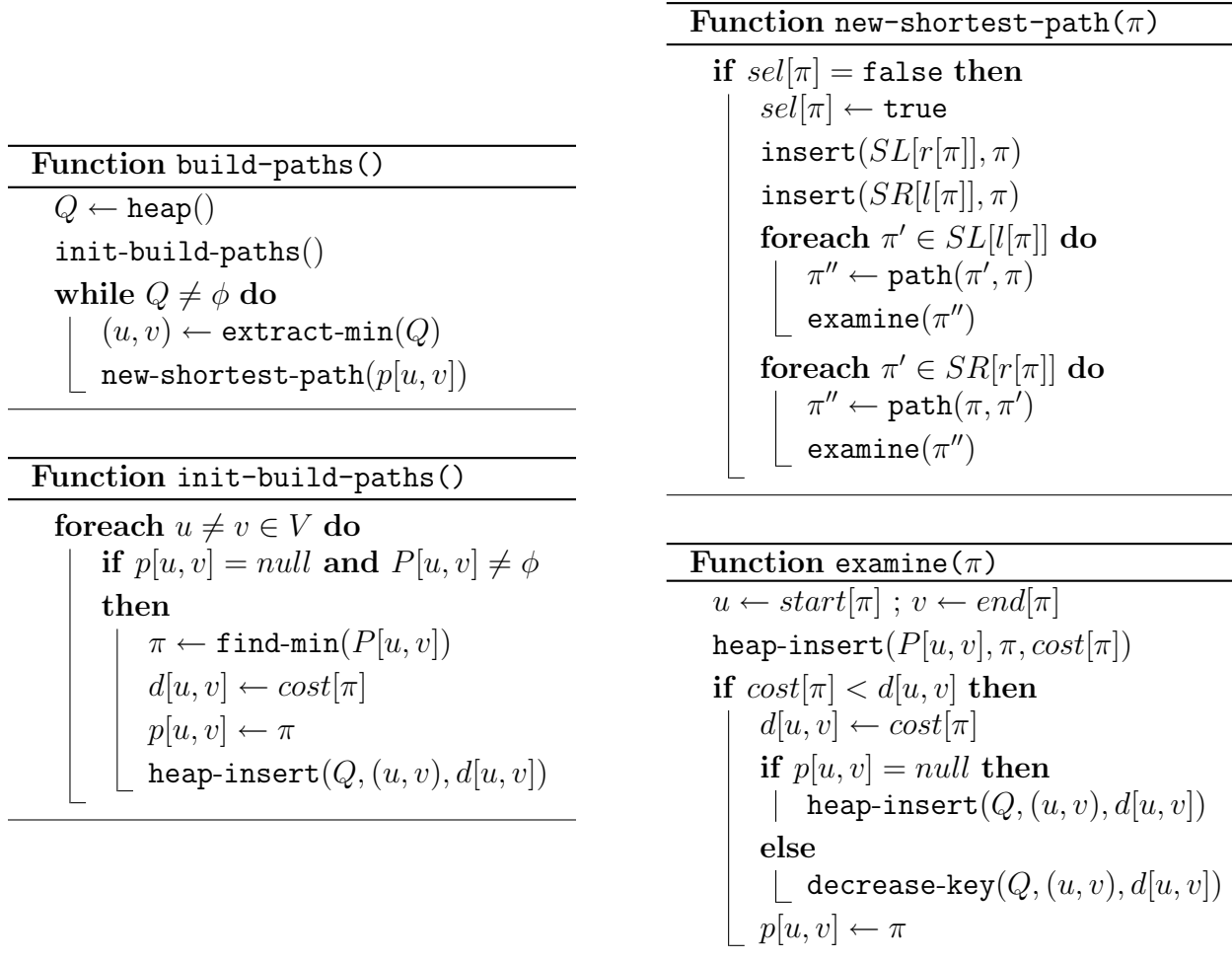


Figure 1: The functions build-paths and new-shortest-path.

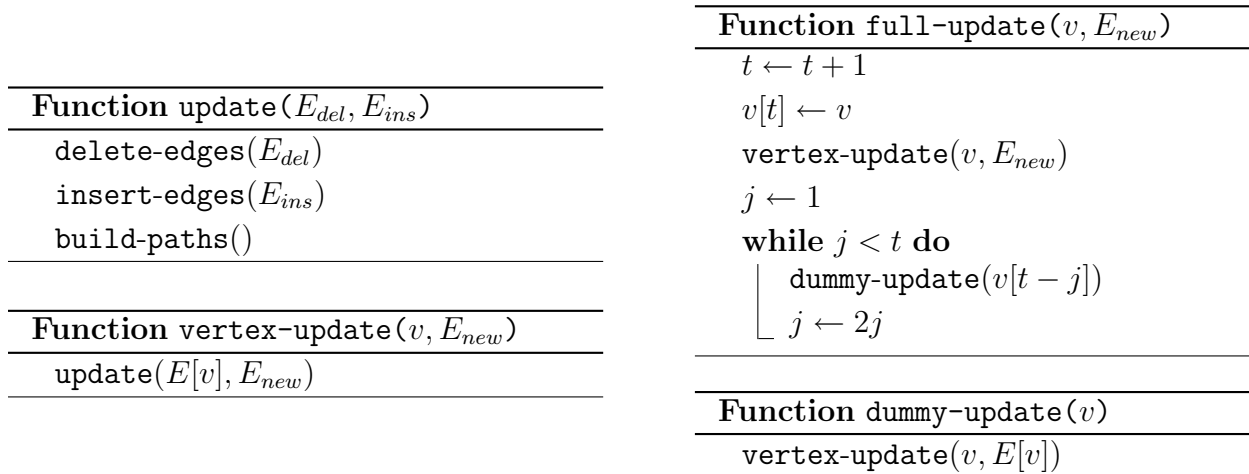


Figure 2: Updating shortest paths.

Function path(v)	Function path($e = (u, v)$)	Function path(π_1, π_2)
$\pi \leftarrow \text{new-path}()$ $l[\pi] \leftarrow \text{null}$ $r[\pi] \leftarrow \text{null}$ $start[\pi] \leftarrow v$ $end[\pi] \leftarrow v$ $first[\pi] \leftarrow \text{null}$ $last[\pi] \leftarrow \text{null}$ $cost[\pi] \leftarrow 0$ $sel[\pi] \leftarrow \text{true}$ $GL[\pi], GR[\pi] \leftarrow \emptyset$ $SL[\pi], SR[\pi] \leftarrow \emptyset$ return π	$\pi \leftarrow \text{new-path}()$ $l[\pi] \leftarrow \pi[u]$ $r[\pi] \leftarrow \pi[v]$ $start[\pi] \leftarrow u$ $end[\pi] \leftarrow v$ $first[\pi] \leftarrow e$ $last[\pi] \leftarrow e$ $cost[\pi] \leftarrow c[e]$ $sel[\pi] \leftarrow \text{false}$ $GL[\pi], GR[\pi] \leftarrow \emptyset$ $SL[\pi], SR[\pi] \leftarrow \emptyset$ $\text{insert}(GL[\pi[v]], \pi)$ $\text{insert}(GR[\pi[u]], \pi)$ return π	if $r[\pi_1] \neq l[\pi_2]$ then error $\pi \leftarrow \text{new-path}()$ $l[\pi] \leftarrow \pi_1$ $r[\pi] \leftarrow \pi_2$ $start[\pi] \leftarrow start[\pi_1]$ $end[\pi] \leftarrow end[\pi_2]$ $first[\pi] \leftarrow first[\pi_1]$ $last[\pi] \leftarrow last[\pi_2]$ $cost[\pi] \leftarrow c[first[\pi]] + cost[\pi_2]$ $sel[\pi] \leftarrow \text{false}$ $GL[\pi], GR[\pi] \leftarrow \emptyset$ $SL[\pi], SR[\pi] \leftarrow \emptyset$ $\text{insert}(GL[\pi_2], \pi)$ $\text{insert}(GR[\pi_1], \pi)$ return π

Figure 3: Generating a new path and inserting it into the path system.

Function remove-path(π)
$u \leftarrow start[\pi] ; v \leftarrow end[\pi]$ if $\pi = p[u, v]$ then $p[u, v] \leftarrow \text{null}$ $d[u, v] \leftarrow \infty$ $\text{heap-delete}(P[u, v], \pi)$ if $l[\pi] \neq \text{null}$ then $\text{delete}(GR[l[\pi]], \pi)$ $\text{delete}(GL[r[\pi]], \pi)$ if $sel[\pi] = \text{true}$ then $\text{delete}(SR[l[\pi]], \pi)$ $\text{delete}(SL[r[\pi]], \pi)$ foreach $\pi' \in GL[\pi] \cup GR[\pi]$ do $\text{remove-path}(\pi')$

Figure 4: Removing a path and all its extensions from the system.