Problem Set no. 4

Given: January 5, 2017
Due: January 19, 2017

Exercise 4.1 (a) Show that the constants $C_1 = C_1(k, b)$ and $C_2 = C_2(k, b)$ of slide 28 can be constructed in $O(\log k)$ time without the use of multiplication (and division). (b) Show that C_1 can be constructed in $O(1)$ if the use of multiplication and division is allowed. (c) Show that if C_1 is available, then C_2 can be constructed in $O(1)$ time with the use of multiplication.

Exercise 4.2 Show that n integers, each of $w/(\log n \log\log n)$-bits, where w is the word size, can be sorted in $O(n)$ time. (Randomization is allowed.)

Exercise 4.3 (a) How much space is used by a straightforward implementation of the $O(n \log\log n)$-time integer sorting algorithm given in class? (b) Show that the space requirement can be reduced to $O(n)$ while maintaining the $O(n \log\log n)$ running time.

Exercise 4.4 Show that for every integer $r \geq 1$, there is a *deterministic* $O(n \log\log n)$-time algorithm for sorting n integers that uses $O(n + 2^{w/r})$ space.

Exercise 4.5 In the description of signature sort, on slide 85, it is claimed that $nq \leq n^2$. (a) Show that this is not always the case. (b) What (minor) change is needed to make the algorithm and its analysis correct?