Problem Set no. 3

Given: December 29, 2016
Due: January 12, 2017

Exercise 3.1 Show that any sorting network is equivalent to a sorting network in *standard form* with the same number of comparators.

Exercise 3.2 (a) Prove, using the 0-1 principle, that Batcher's Bitonic Sorter, for $n = 2^k$, sorts any *cyclic* shift of a bitonic sequence. Justify the use of the 0-1 principle. (b) Prove that Batcher's Bitonic Sorter, for $n = 2^k$, sorts any *cyclic* shift of a bitonic sequence *without* using the 0-1 principle.

Exercise 3.3 (a) Show that if items fed into k input wires may end up in a given output wire, then the depth of the network is at least $\lg k$. (b) Show that the depth of an *halver* of n items must be at least $\lg(\frac{n}{2} + 1)$. (c) Show that the depth of network that merges two sorted sequences of n items each is at least $\lg(2n)$.

Exercise 3.4 Prove the following claims needed in the analysis of the AKS network: (a) Show that the "ideal" distribution considered in slide 72 does indeed exist. (More precisely, let C be any node in the tree. Prove that if we sum up the current sizes of all the descendants of C and the specified fractions of the current sizes of the ancestors of C, we get exactly the number of items native to C.) (b) Show that at the leaves we always have $m \le 2\lfloor \lambda b \rfloor + 1$, so all items at the leaves are sent up. (c) Show that if at a node B we have $b < A$, then all nodes above the level of B are empty and m, the number of items in B, is even.

Exercise 3.5 In the analysis of the AKS network we assumed that each node uses a $(\lambda', \epsilon, \epsilon)$-separator, where $\frac{\lambda' m}{2} = \lfloor \lambda b \rfloor$. Suppose that we use now a $(\lambda', \epsilon, \epsilon_0)$-separator, where $\epsilon_0 < \epsilon$. Which of the required inequalities on slide 75 change and to what?