**Algorithm: Forward algorithm for profile HMMs**

Initialisation: $f_{\mathrm{M}_0}(0) = 1$.

Recursion:
$$f_{\mathrm{M}_k}(i) = e_{\mathrm{M}_k}(i)[f_{\mathrm{M}_{k-1}}(i-1)a_{\mathrm{M}_{k-1}\mathrm{M}_k} + f_{\mathrm{I}_{k-1}}(i-1)a_{\mathrm{I}_{k-1}\mathrm{M}_k} + f_{\mathrm{D}_{k-1}}(i-1)a_{\mathrm{D}_{k-1}\mathrm{M}_k}];$$
$$f_{\mathrm{I}_k}(i) = e_{\mathrm{I}_k}(i)[f_{\mathrm{M}_k}(i-1)a_{\mathrm{M}_k\mathrm{I}_k} + f_{\mathrm{I}_k}(i-1)a_{\mathrm{I}_k\mathrm{I}_k} + f_{\mathrm{D}_k}(i-1)a_{\mathrm{D}_k\mathrm{I}_k}];$$
$$f_{\mathrm{D}_k}(i) = f_{\mathrm{M}_{k-1}}(i)a_{\mathrm{M}_{k-1}\mathrm{D}_k} + f_{\mathrm{I}_{k-1}}(i)a_{\mathrm{I}_{k-1}\mathrm{D}_k} + f_{\mathrm{D}_{k-1}}(i)a_{\mathrm{D}_{k-1}\mathrm{D}_k}.$$

Termination:
$$f_{\mathrm{M}_{M+1}}(L+1) = f_{\mathrm{M}_M}(L)a_{\mathrm{M}_M\mathrm{M}_{M+1}} + f_{\mathrm{I}_M}(L)a_{\mathrm{I}_M\mathrm{M}_{M+1}} + f_{\mathrm{D}_M}(L)a_{\mathrm{D}_M\mathrm{M}_{M+1}}.$$
◁

**Algorithm: Backward algorithm for profile HMMs**

Initialisation:
$$b_{\mathrm{M}_{M+1}}(L+1) = 1;$$
$$b_{\mathrm{M}_M}(L) = a_{\mathrm{M}_M\mathrm{M}_{M+1}};$$
$$b_{\mathrm{I}_M}(L) = a_{\mathrm{I}_M\mathrm{M}_{M+1}};$$
$$b_{\mathrm{D}_M}(L) = a_{\mathrm{D}_M\mathrm{M}_{M+1}}.$$

Recursion:
$$b_{\mathrm{M}_k}(i) = b_{\mathrm{M}_{k+1}}(i+1)a_{\mathrm{M}_k\mathrm{M}_{k+1}}e_{\mathrm{M}_{k+1}}(x_{i+1}) + b_{\mathrm{I}_k}(i+1)a_{\mathrm{M}_k\mathrm{I}_k}e_{\mathrm{I}_k}(x_{i+1}) + b_{\mathrm{D}_{k+1}}(i)a_{\mathrm{M}_k\mathrm{D}_{k+1}};$$
$$b_{\mathrm{I}_k}(i) = b_{\mathrm{M}_{k+1}}(i+1)a_{\mathrm{I}_k\mathrm{M}_{k+1}}e_{\mathrm{M}_{k+1}}(x_{i+1}) + b_{\mathrm{I}_k}(i+1)a_{\mathrm{I}_k\mathrm{I}_k}e_{\mathrm{I}_k}(x_{i+1}) + b_{\mathrm{D}_{k+1}}(i)a_{\mathrm{I}_k\mathrm{D}_{k+1}};$$
$$b_{\mathrm{D}_k}(i) = b_{\mathrm{M}_{k+1}}(i+1)a_{\mathrm{D}_k\mathrm{M}_{k+1}}e_{\mathrm{M}_{k+1}}(x_{i+1}) + b_{\mathrm{I}_k}(i+1)a_{\mathrm{D}_k\mathrm{I}_k}e_{\mathrm{I}_k}(x_{i+1}) + b_{\mathrm{D}_{k+1}}(i)a_{\mathrm{D}_k\mathrm{D}_{k+1}}.$$
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The forward and backward variables can then be combined to re-estimate emission and transition probability parameters as follows:

**Algorithm: Baum–Welch re-estimation equations for profile HMMs**

Expected emission counts from sequence $x$:

$$E_{\mathrm{M}_k}(a) = \frac{1}{P(x)} \sum_{i|x_i=a} f_{\mathrm{M}_k}(i)b_{\mathrm{M}_k}(i);$$
$$E_{\mathrm{I}_k}(a) = \frac{1}{P(x)} \sum_{i|x_i=a} f_{\mathrm{I}_k}(i)b_{\mathrm{I}_k}(i).$$

Expected transition counts from sequence $x$:

$$A_{\mathrm{X}_k\mathrm{M}_{k+1}} = \frac{1}{P(x)} \sum_i f_{\mathrm{X}_k}(i)a_{\mathrm{X}_k\mathrm{M}_{k+1}}e_{\mathrm{M}_{k+1}}(x_{i+1})b_{\mathrm{M}_{k+1}}(i+1);$$
$$A_{\mathrm{X}_k\mathrm{I}_k} = \frac{1}{P(x)} \sum_i f_{\mathrm{X}_k}(i)a_{\mathrm{X}_k\mathrm{I}_k}e_{\mathrm{I}_k}(x_{i+1})b_{\mathrm{I}_k}(i+1);$$
$$A_{\mathrm{X}_k\mathrm{D}_{k+1}} = \frac{1}{P(x)} \sum_i f_{\mathrm{X}_k}(i)a_{\mathrm{X}_k\mathrm{D}_{k+1}}b_{\mathrm{D}_{k+1}}(i).$$