Simple Auctions for Agents with Complements

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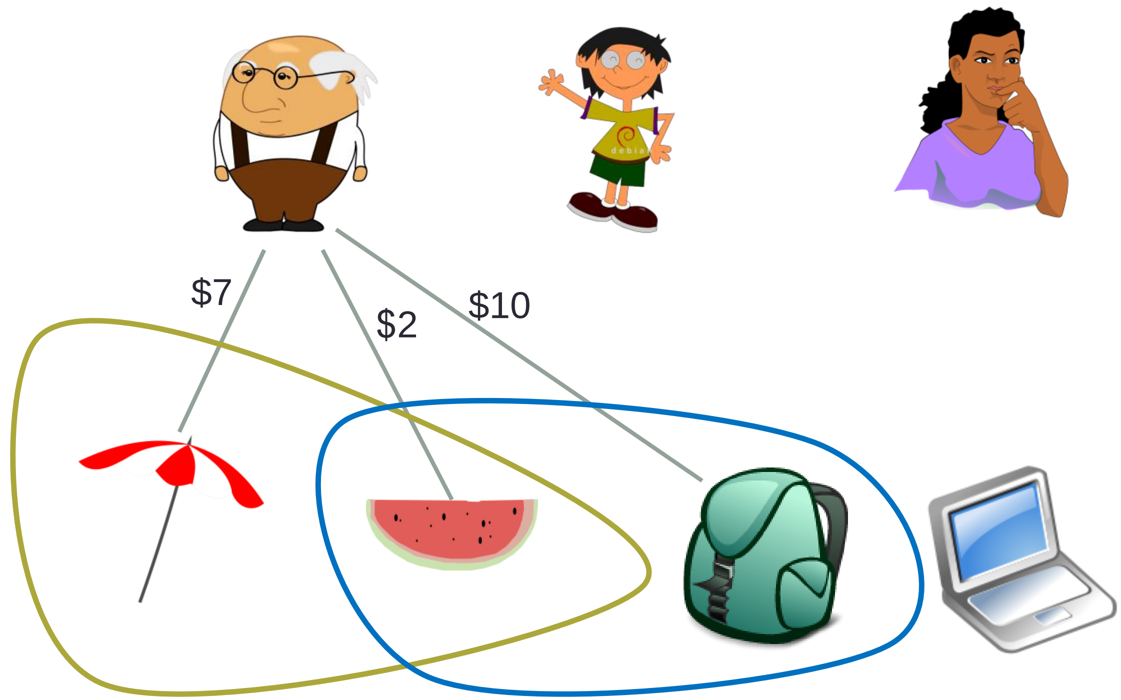
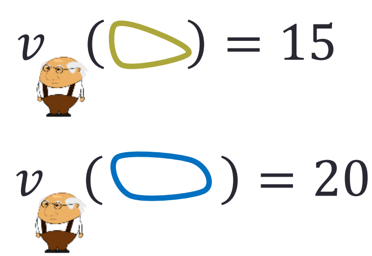
Combinatorial Auctions

- n bidders
- m items

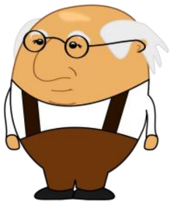




Combinatorial Auctions

Objective: Designing combinatorial auctions with good properties, e.g.:

- Efficient with respect to social welfare $\sum_i v_i\,(S_i)$.
- Computationally tractable.
- Simple design.
- Truthful.
- (Or at least) simple to strategize.



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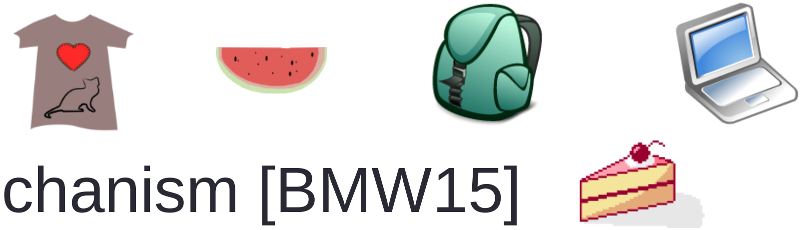
Outline

- Single Bid Auction
- Complements
  - Existing hierarchies
  - New hierarchy
- Results
  - POA of the single bid auction degrades gracefully with the level of complementarity in our new hierarchy.
- Summary + Future work

The Single-Bid Auction [DMSW15]

Simple design!

- X is the set of items.
- Each bidder submits a single bid b_i
- Auctioneer sorts bidders by highest bid
- For each bidder i:
  - Let i choose $X_i \subseteq X$
  - i pays $p_i = b_i \cdot |X_i|$
  - $X \leftarrow X \setminus X_i$



Simple in a formal sense:
A priori learnable interpolation mechanism [BMW15]

Single bid auctions for complement-free bidders

Price of anarchy = $\dfrac{SW(OPT)}{SW(worst\ eq)}$

Theorem [DMSW15] : The single-bid auction has a price of anarchy of at most $\frac{e}{e-1}\log m$ for agents with subadditive valuations.

A valuation is subadditive (complement-free) if $v(T \cup S) \leq v(T) + v(S)$ for any two bundles T, S.

Proof via smoothness [ST13] → Mixed Nash, Bayes-Nash, correlated, coarse correlated.

Standard regret minimization algorithms run in poly time/space, and converge to correlated equilibrium.

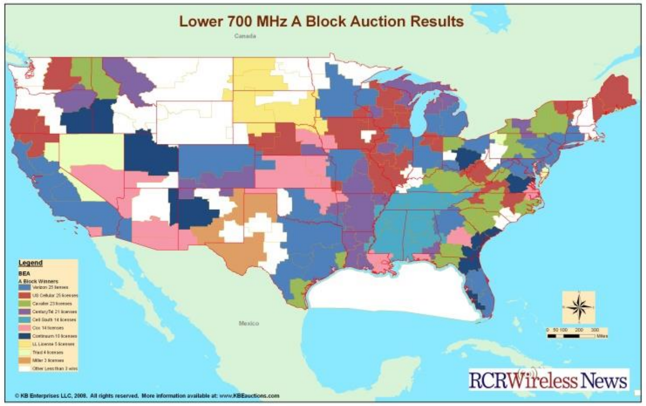
$\mathrm{POA} = \Theta(m)$ for general valuations

What About Complements?

Items with complements are very natural and appear in many situations

Cloud computing

FCC Spectrum



Airport landing slots

Shopping

Related works on complements

Truthful:

- Universally truthful mechanisms [DNS06], [D07]:
  - $O(\sqrt{m})$ for general valuations.
- Truthful in expectation mechanisms [LS05], [ABDR12]:
  - $O(\sqrt{m})$ for general valuations.
  - $O(\log^k m)$ for PH-k valuations.
- Posted prices [FGL15]:
  - $O(k)$ for MPH-k valuations (Bayesian setting).

Non-truthful:

- Simultaneous single item auctions [FFIILS15]:
  - $O(k)$ for MPH-k Valuations.
- Greedy auctions [BL10]:
  - $O(\sqrt{m})$ for general valuations.

Welfare maximization

- (d) for supermodular degree d valuations [FI13].

How do we study complements?

Not much to do in the general case:

- Welfare is hard to approximate within $\sqrt{m}$, even for single-minded valuations [LOS02],[S02].

Restricted complements

Rank by the “size” of the complementarity.

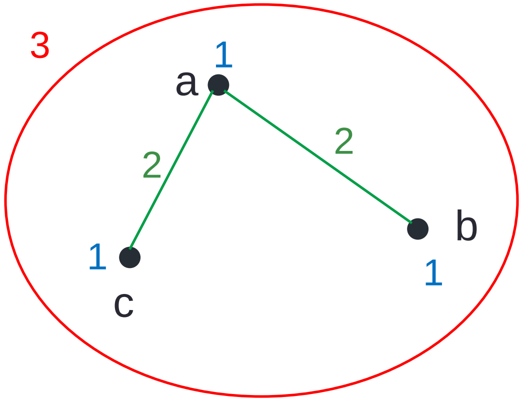
Restricted Complements

Positive Hypergraph (PH) [ABDR12] :

- The sets in $2^{[m]}$ are (hyper-)edges with weights $w_e \geq 0$
- The value of a set is the sum of contained hyperedges.

$$v(S) = \sum_{T \subseteq S} w_T$$

Level k: Edges with **positive** weight are of size at most k.



$$v(\{a,b\}) = 1 + 1 + 2 = 4$$

$$v(\{a,b,c\}) = 1 + 1 + 1 + 2 + 2 + 3 = 10$$

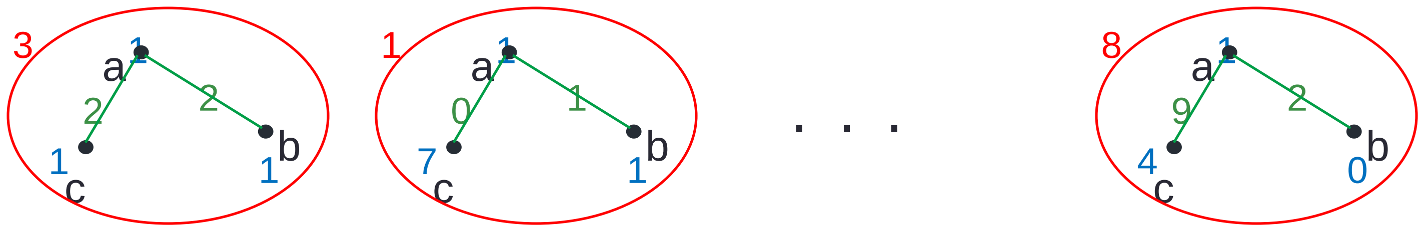
No representation for “real” substitutes.

Restricted Complements

Maximum over Positive Hypergraph (MPH) [FFIILS15] :

- A collection of (non-negative) weighted hypergraphs.

$$w^1, w^2, \dots, w^\ell : 2^{[m]} \to \mathbb{R}_{\geq 0}$$



- The value of a set is the maximum value within the collection:

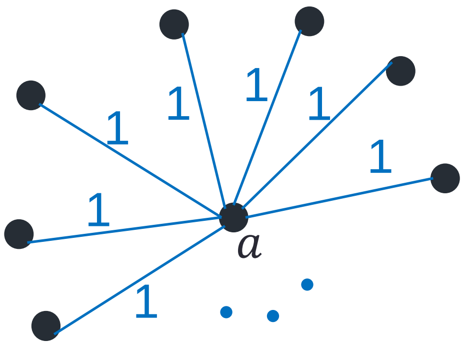
$$v(S) = \max_j \sum_{T \subseteq S} w_T^j$$

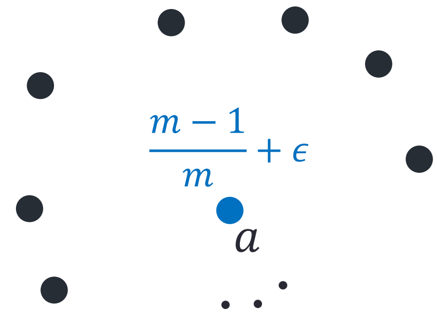
Level k: Edges with positive weight are of size at most k

Encompasses all valuations!

Performance of the single bid auction

$POA = \Omega(m)$ already for PH-2 valuations, with two bidders:





v  $(S: a \in S) = |S| - 1$

v  $(S: a \in S) = \frac{m-1}{m} + \epsilon$

- In any pure Nash equilibrium -  doesn't get anything.
  -  will not bid more than $\frac{m-1}{m}$.
  -  can always outbid 

Performance of the single bid auction

The single bid auction has terrible performance in the presence of complements.

Or…

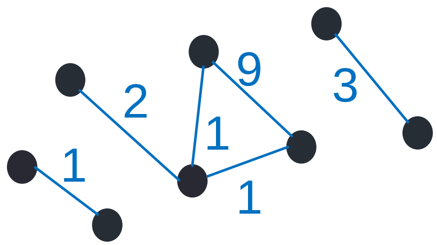
We need to view complements from a different lens.

Idea: Bound the number of neighbors

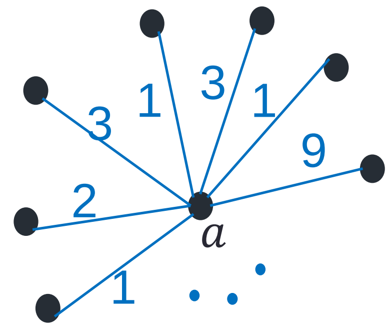
Positive Supermodular (PS) of level d:

Each item may be a complement to at most d items

Level 3:



Level $m-1$:



In other words: Each item shares a positive weighted edge with at most d other items.

MPS-d: Maximum over a collection of PS-d valuations.

Encompasses all valuations!

Main Result

The single-bid auction has a price of anarchy of at most $O(d^2 \log \frac{m}{d})$ for agents with MPS-d valuations.

Proof via smoothness [ST13] → Mixed Nash, Bayes-Nash, correlated, coarse correlated

Upper bound of $O(d \log m)$ for an interesting special case (edge size ≤ 2)

Lower bound of d on the price of anarchy (and stability)

Proof Outline

Pointwise β-approximation [DMSW15]:
If a class of valuations V' is (λ, μ)-smooth, and class V is β-apx. by V', then V is $(\frac{\lambda}{\beta}, \mu)$-smooth.

Step 1: capture complementarities in a $\left(\frac{1-e^{-d}}{d}, 1\right)$-smooth class (we call this class d-constraint homogeneous).

Step 2: Show pointwise approximation of PS-d:

Lemma: Positive Supermodular d valuations are $(d+2) \cdot H_{\frac{m}{d+1}}$ -pointwise approximated by $(d+1)$-constraint homogenous valuations.

[Maximum over a collection comes for free]

Simple mechanisms for general valuations

For general valuations, the POA of the single bid auction is $\Omega(m)$.

An improved, still simple mechanism:

Single bid auction

Single bid for the grand bundle

Randomize →

$POA = O(\sqrt{m})$ for general valuations

(And it is an a priori learnable interpolation mechanism)

Summary

- Shown a hierarchy (MPS-d) that expresses how complementarities affect the POA in single bid auctions:
  - Upper bound $O(d^2 \log\frac{m}{d})$, lower bound d.
- Improved POA for an interesting special case (hyperedges of size ≤2)
  - Upper bound $O(d \log m)$, lower bound d.
- Provided a simple mechanism that achieves $O(\sqrt{m})$ price of anarchy.

Future work

- Close gap between lower (d) and upper ($O(d^2 \log\frac{m}{d})$) bounds for the single bid auction with restricted complements.
- Achieve meaningful bounds for restricted complementes in other settings, e.g.:
  - The supermodular degree [FI13].
  - Consider universally truthful mechanisms.
  - …

Thank you!