Separating the Wheat from the Chaff:

Understanding (In-)Completeness of Proof Mechanisms for Separation Logic with Inductive Definitions

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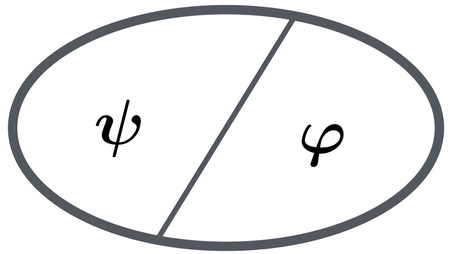

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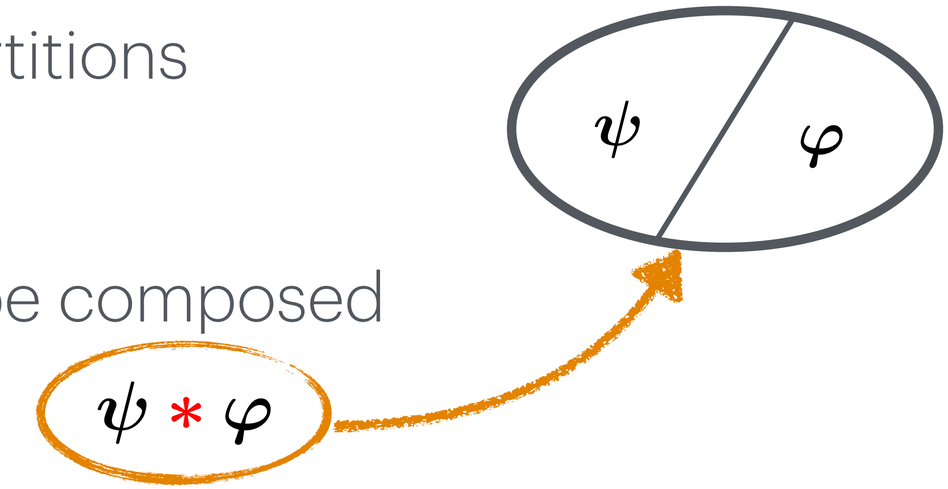

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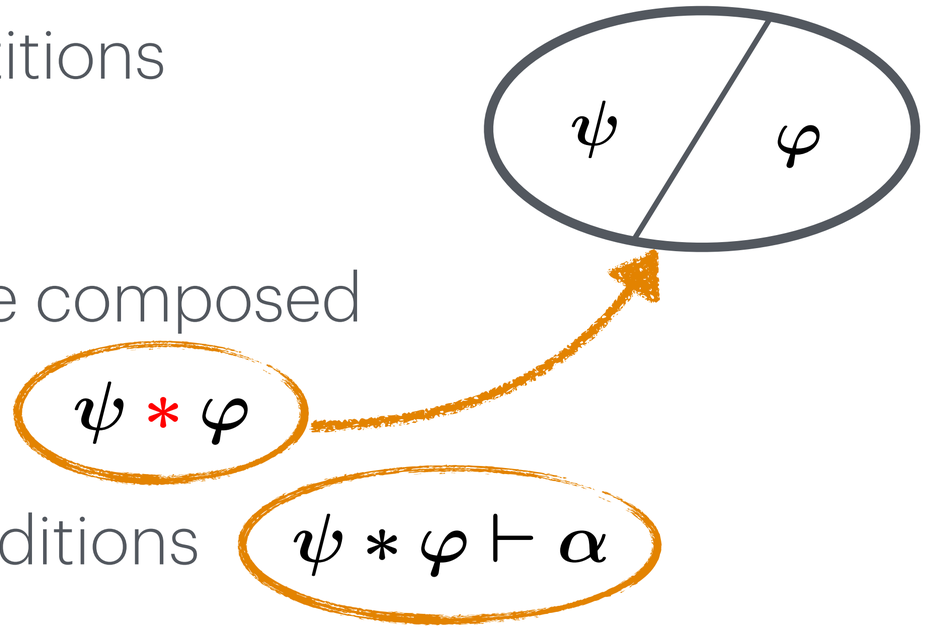

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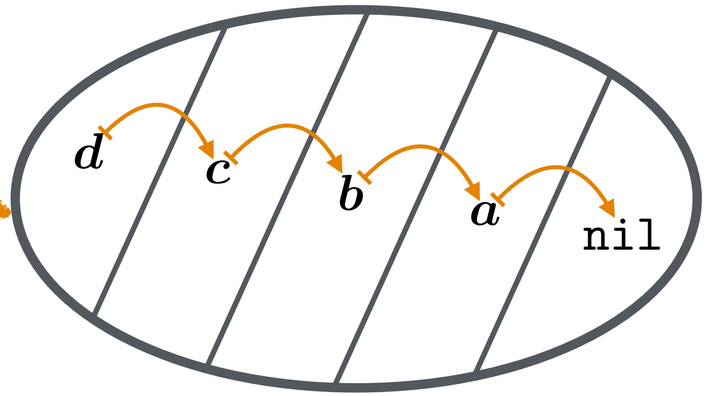

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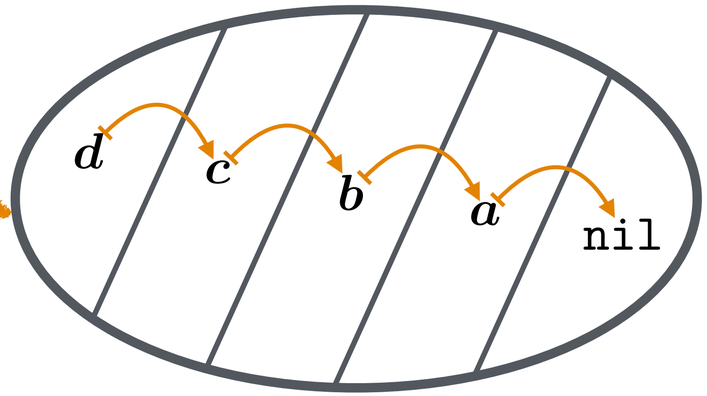

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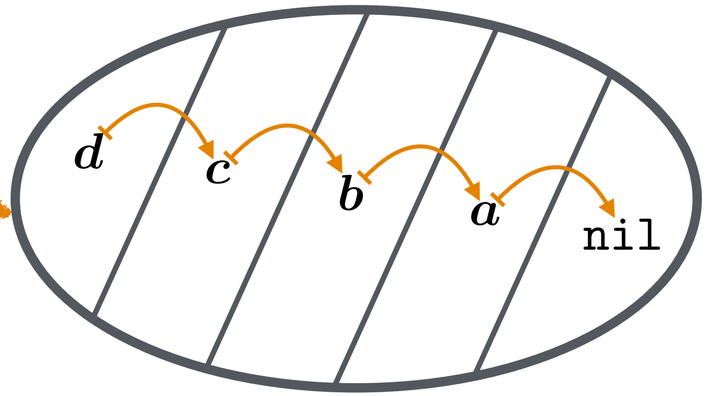

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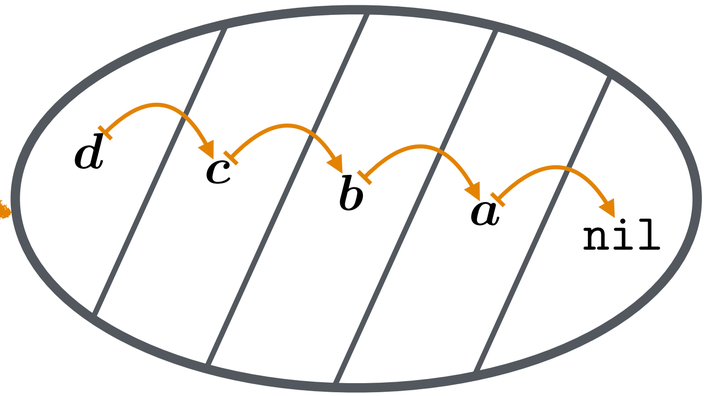

- SLID is very expressive but incomplete

- Prominent proof mechanism: fold/unfold
  - Successful in some cases but fails in others

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Why? And more generally, when?

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- When fails? When *rogue* counter-models exist
  - Rogue meaning models not considered by SLID semantics

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- Conclusion

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Theorem 5.4. Fold/unfold is sound for WSL

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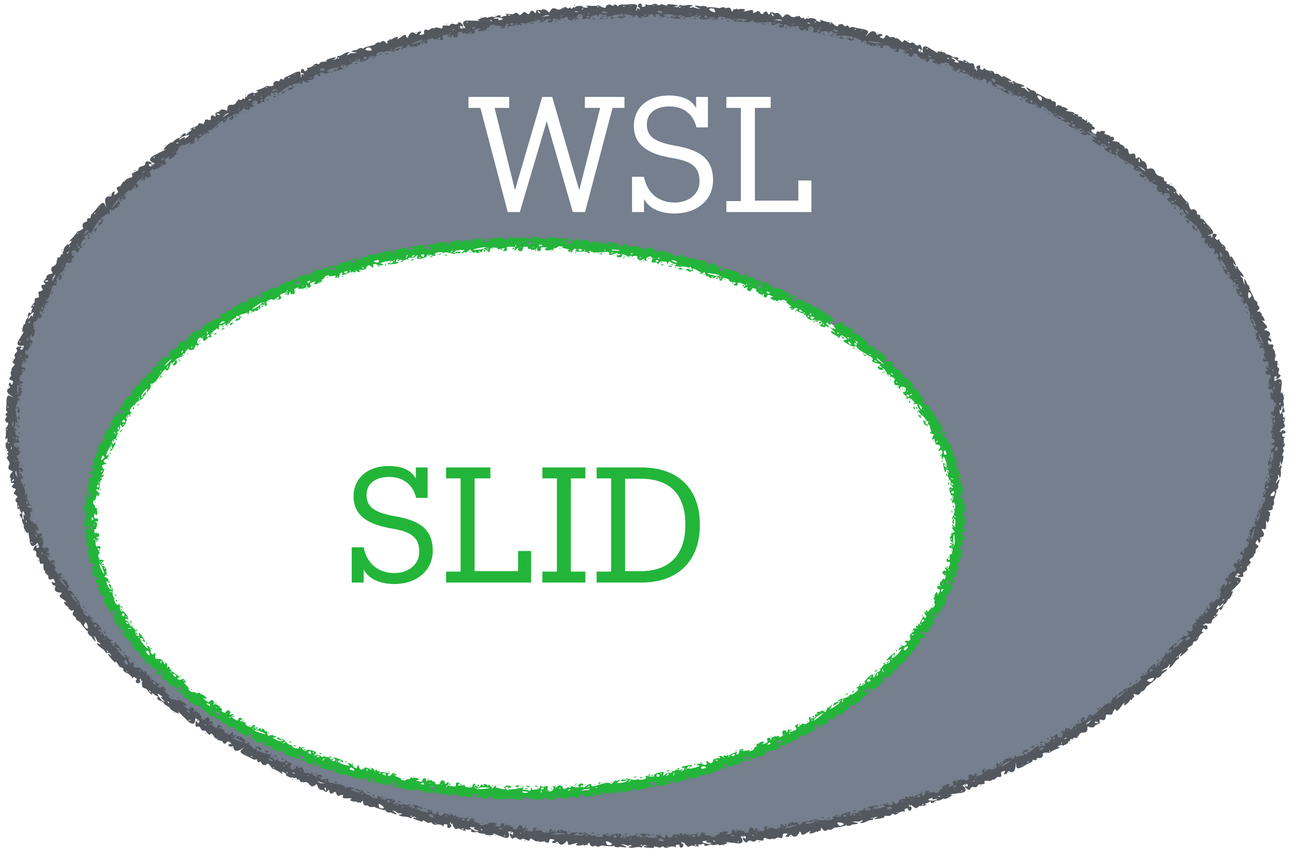

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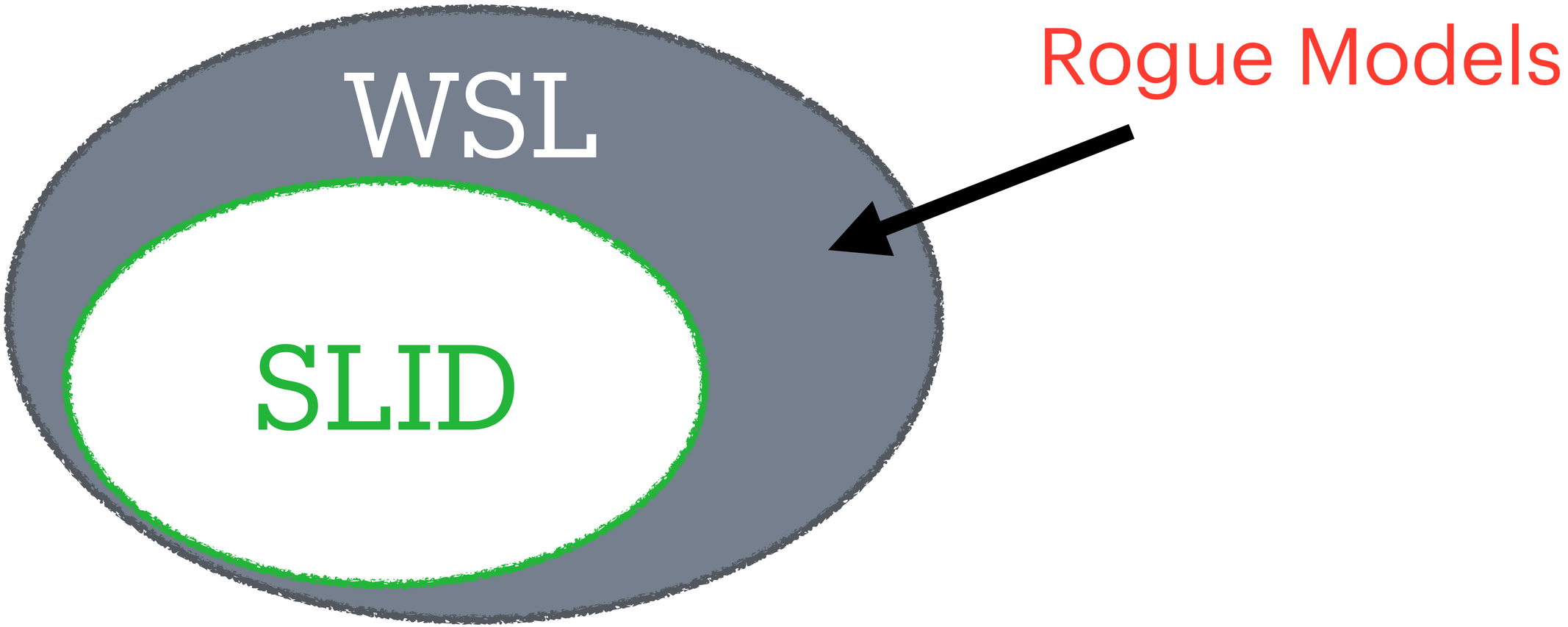

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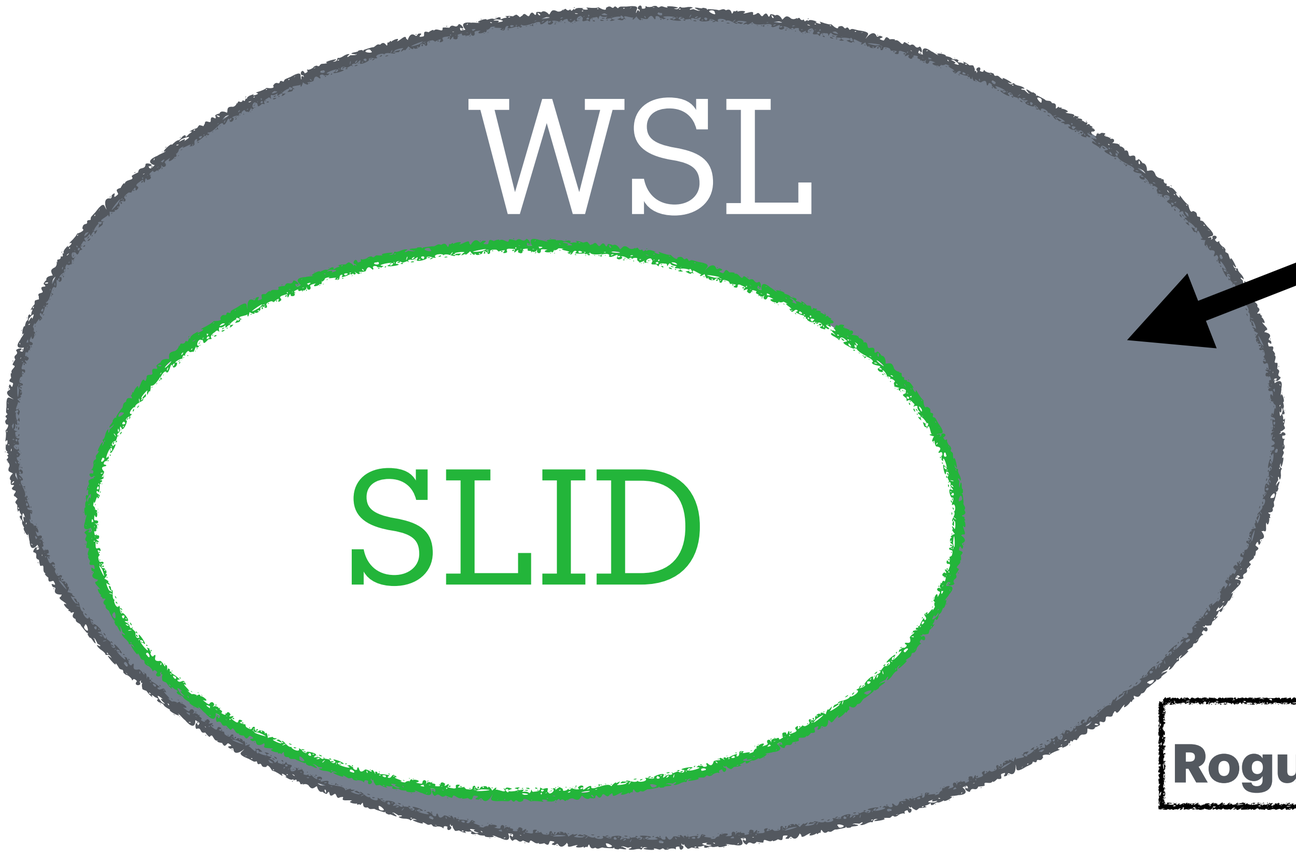

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Rogue counter-models ⇒ fold/unfold failures

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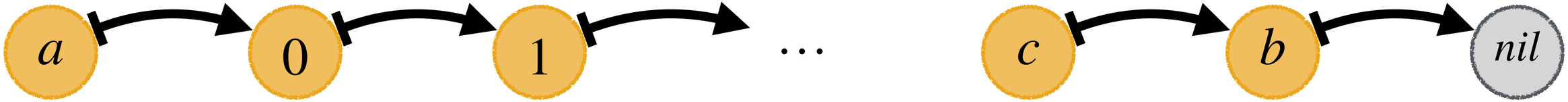

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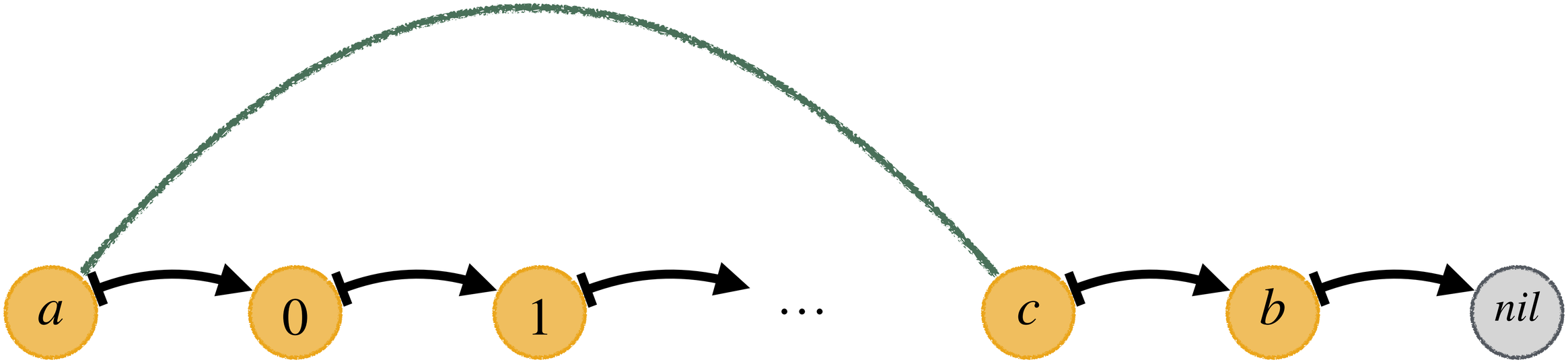

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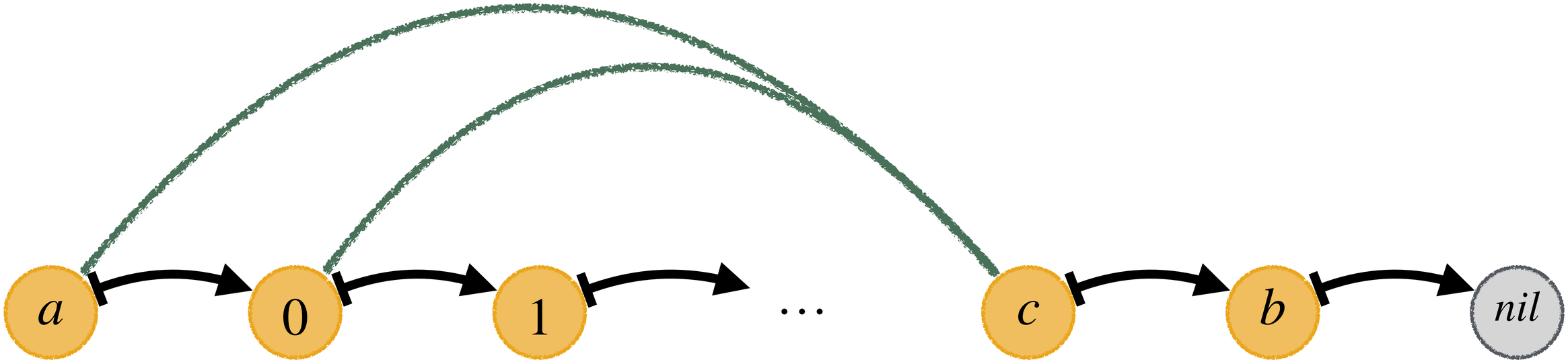

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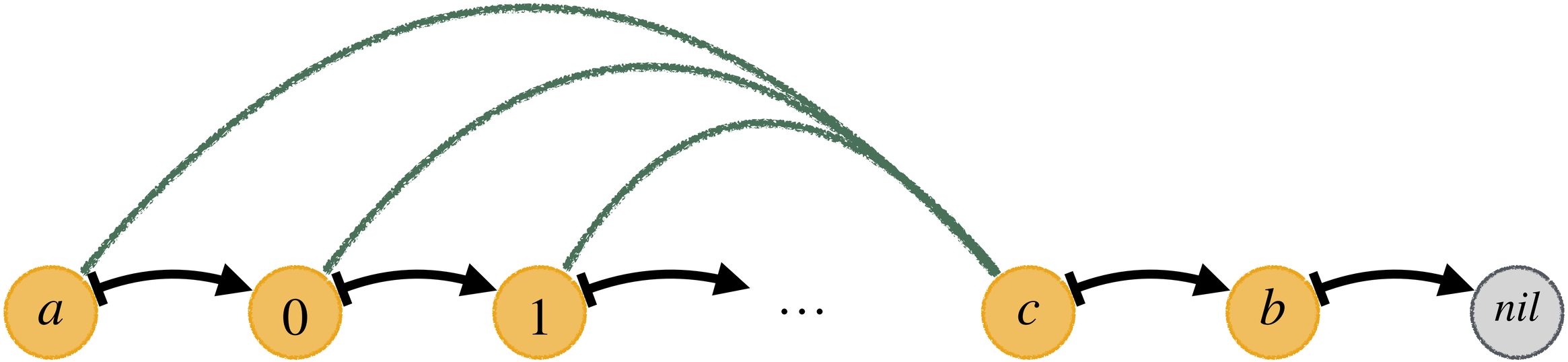

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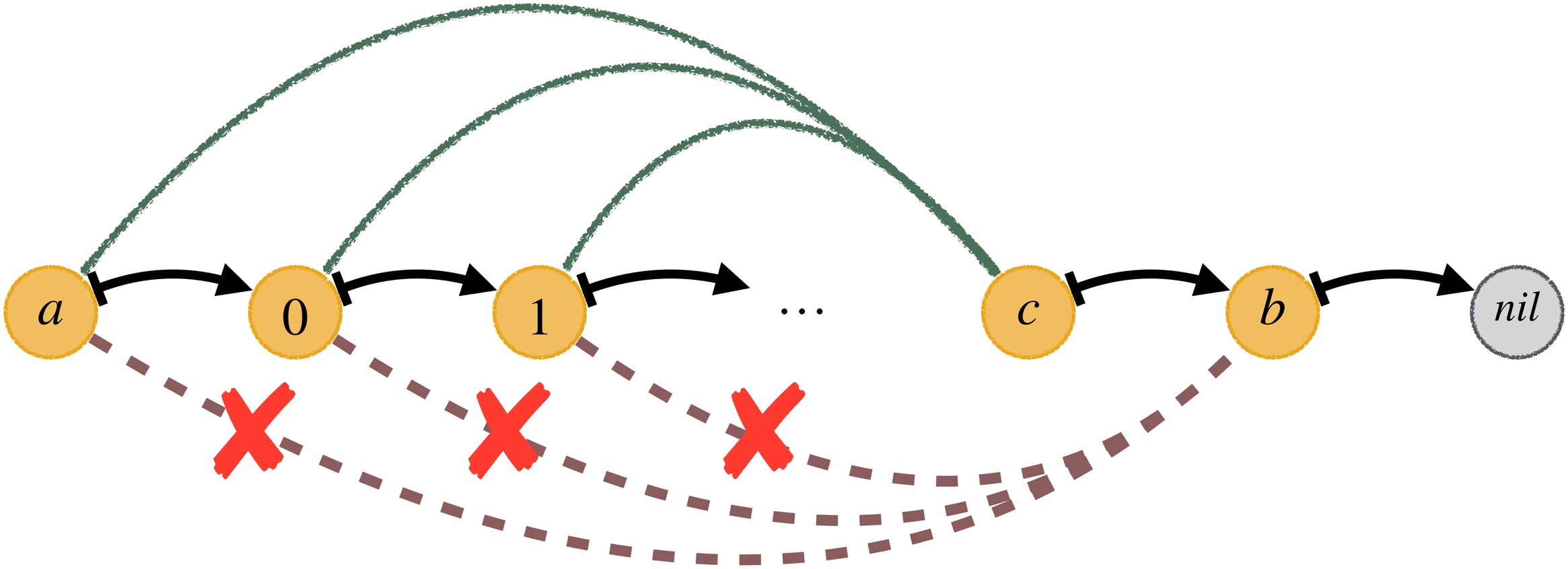

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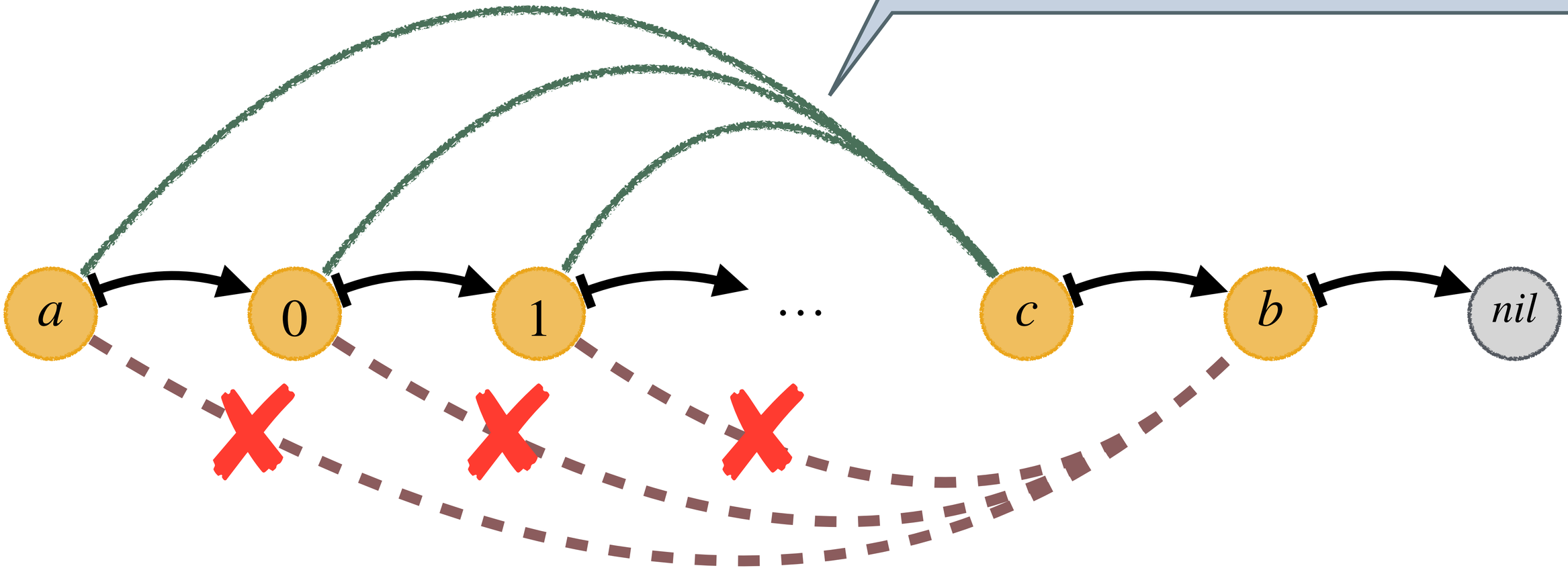

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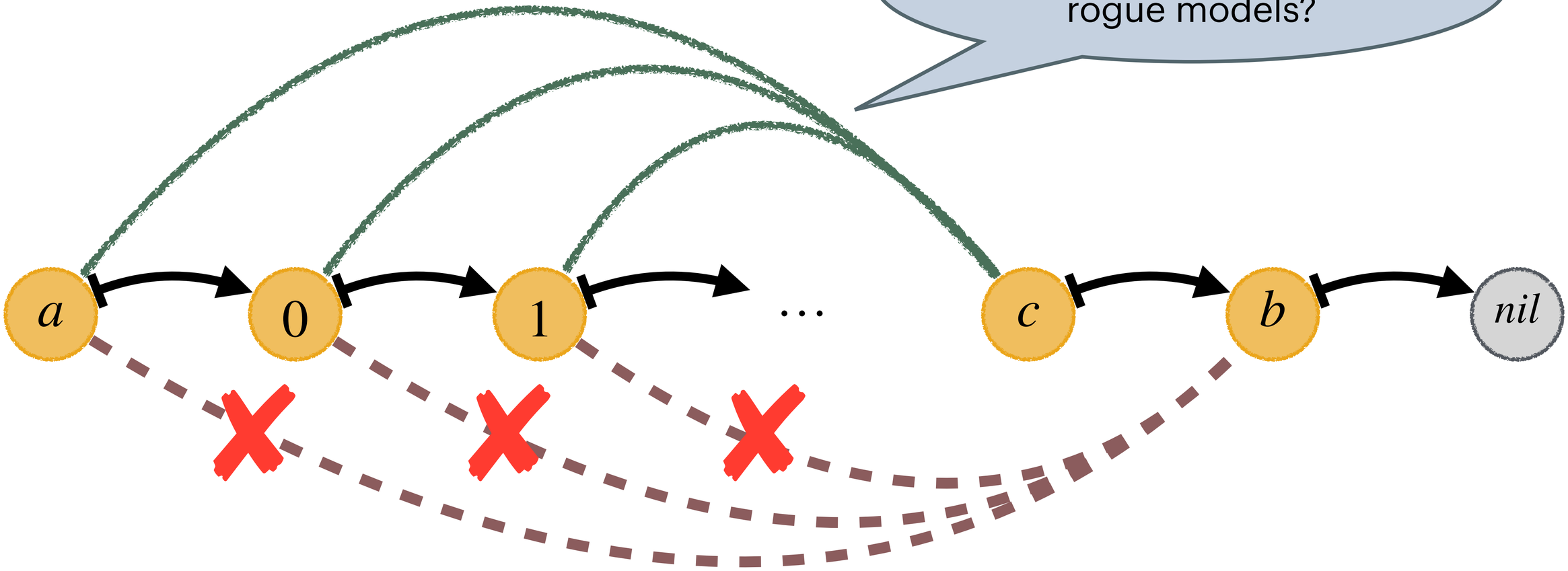

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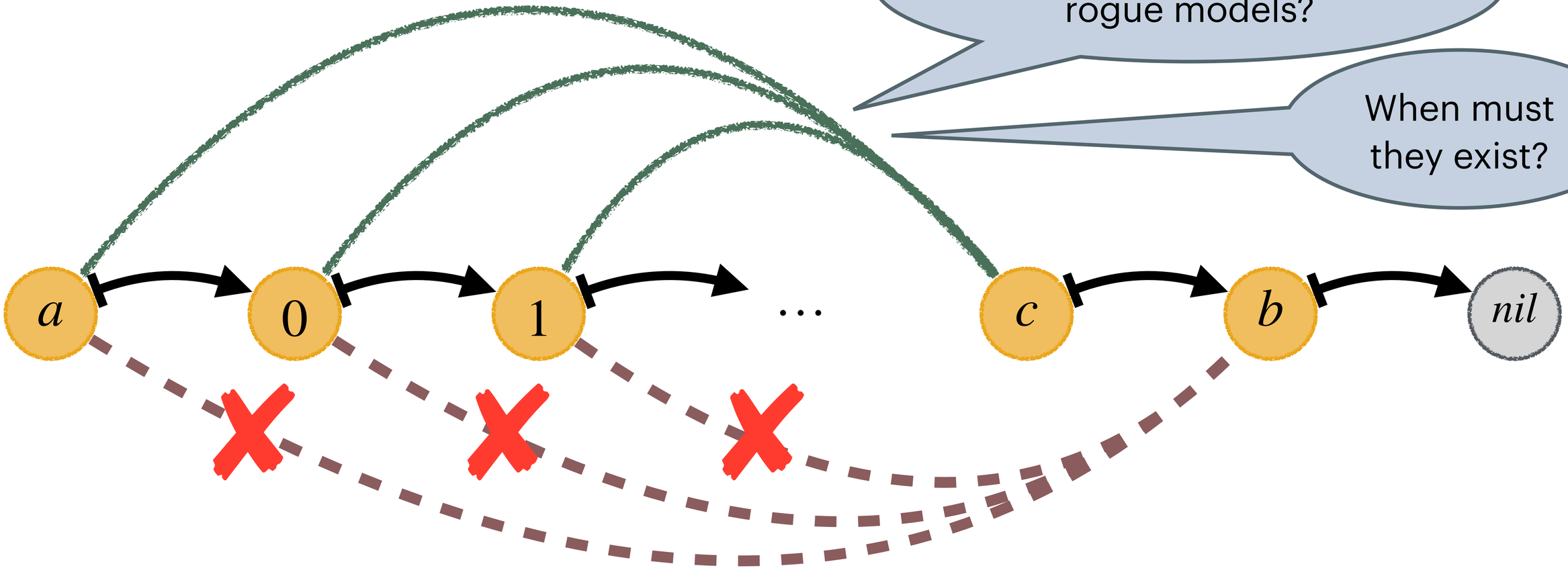

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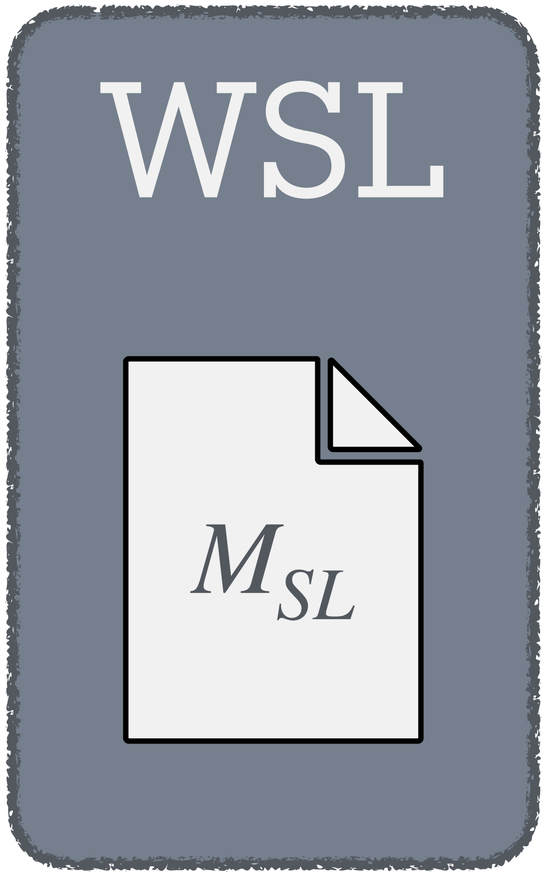

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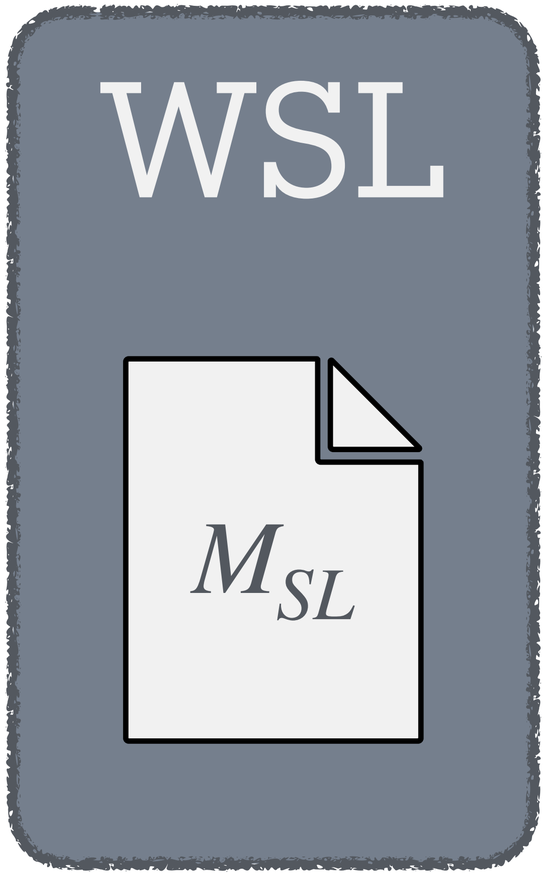

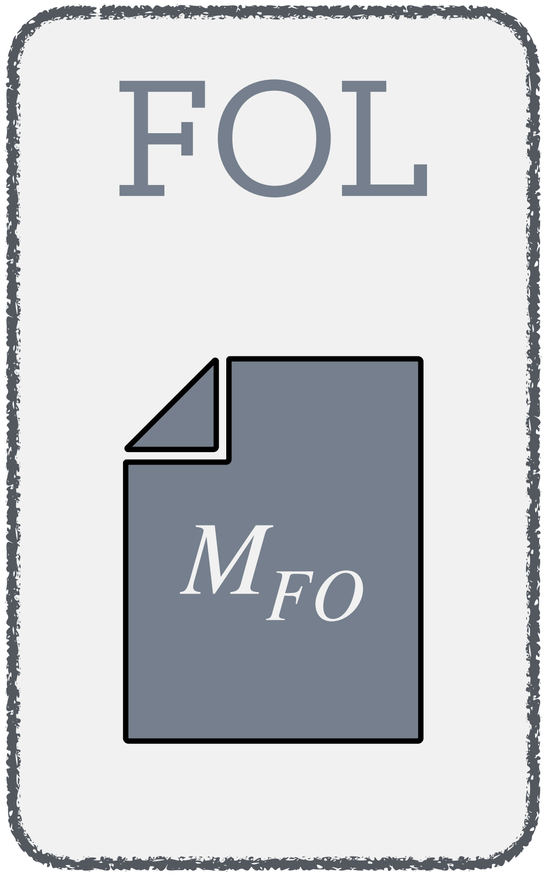

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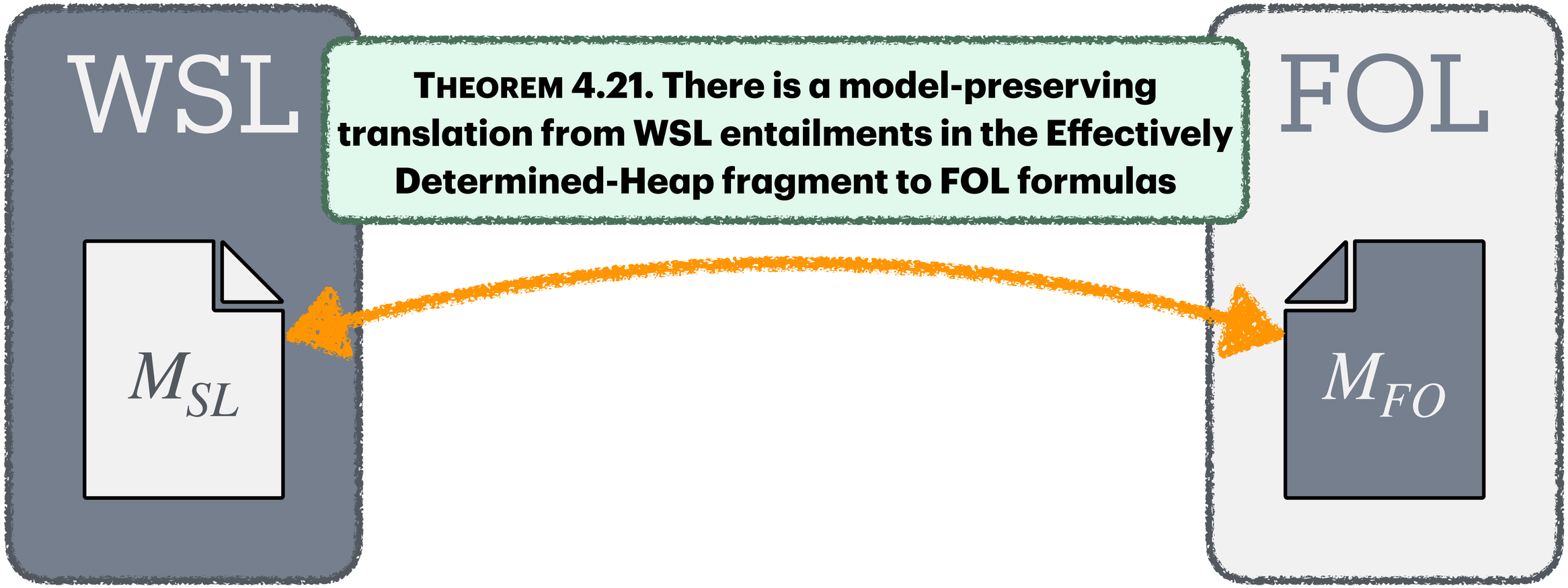

Rogue Models as Symbolic Structures

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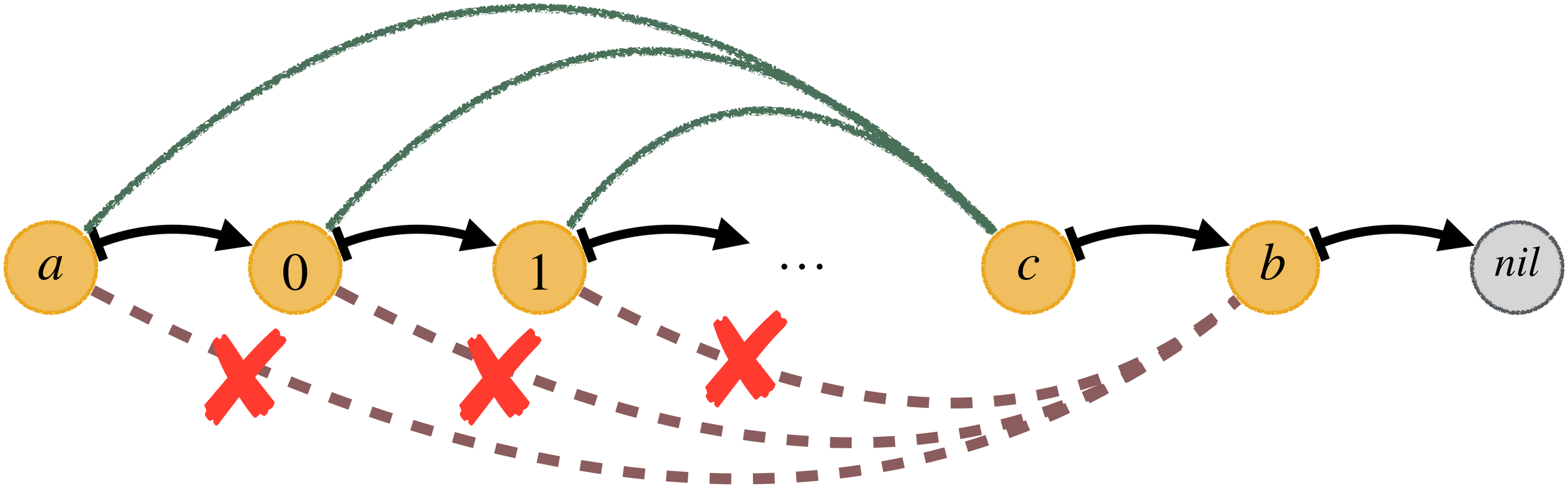

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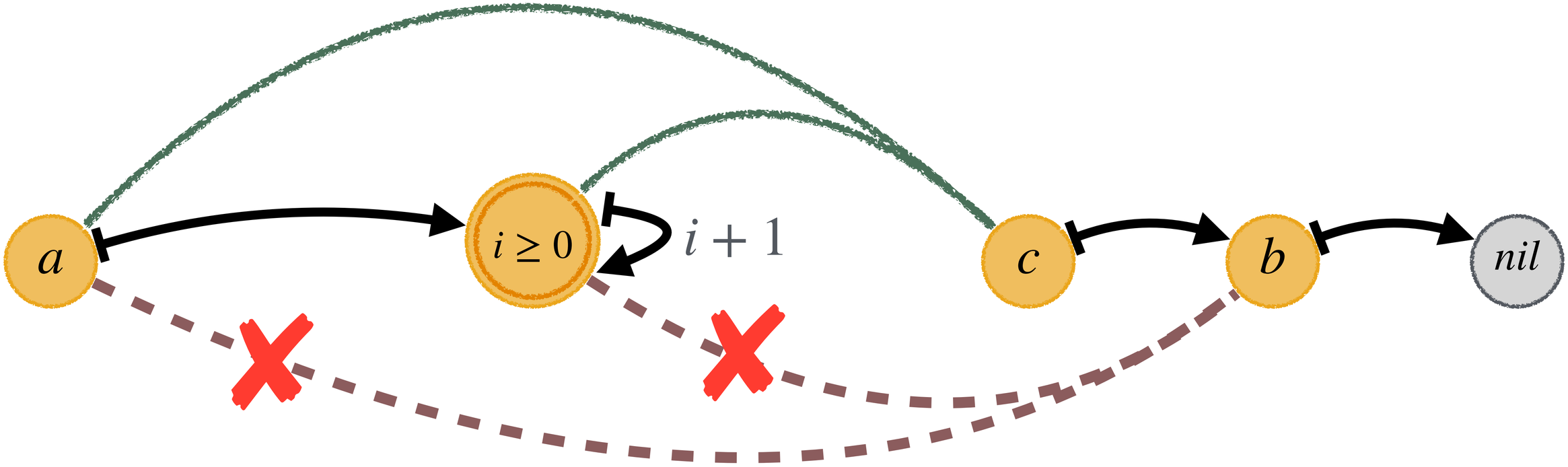

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THEOREM 4.21

Set of quantified formulas is unsatisfiable in FOL

Rogue Models of WSL and Fold/Unfold Failures

Do rogue models of WSL capture all fold/unfold failures?

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HERBRAND'S THEOREM

COMPACTNESS

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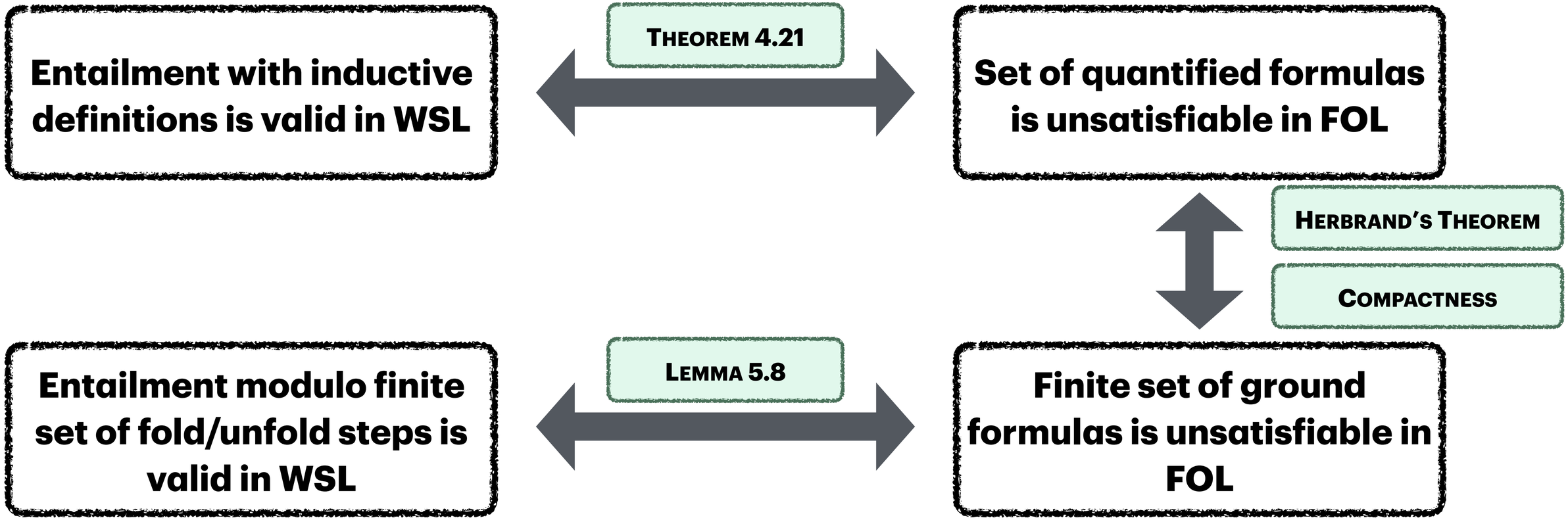

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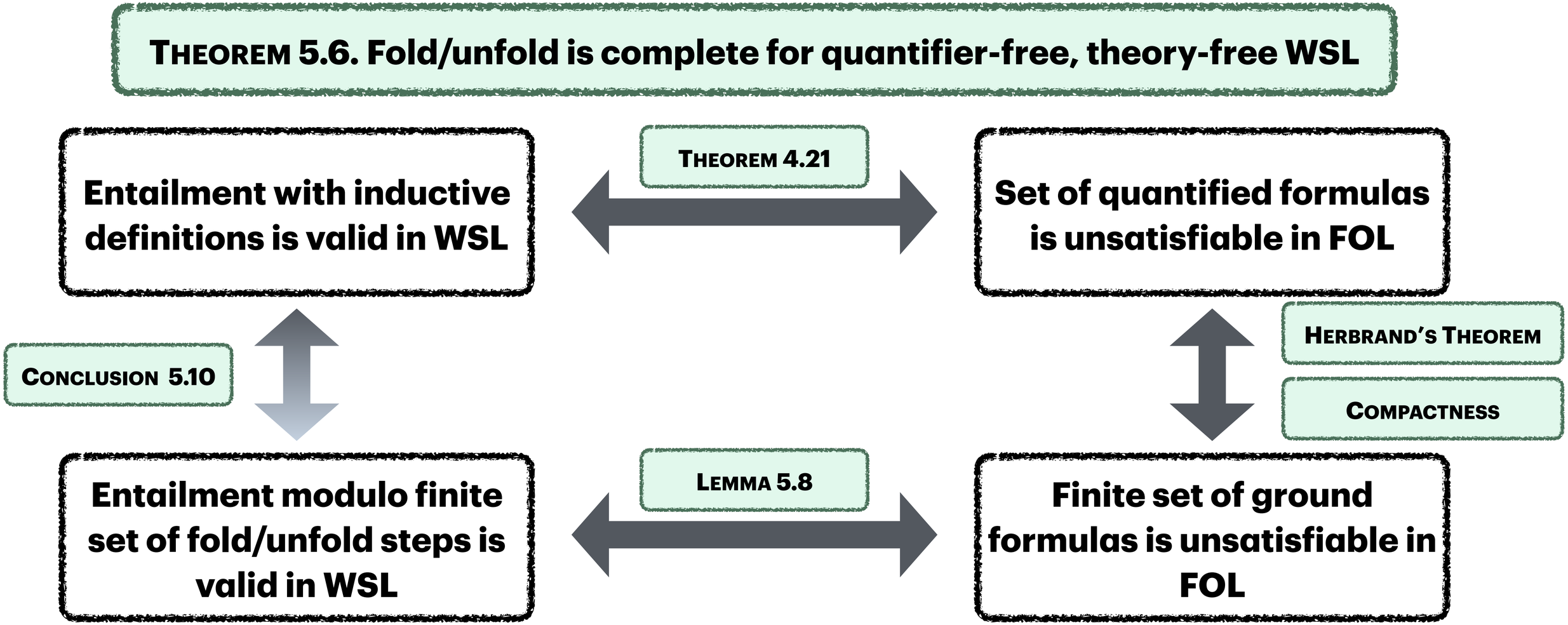

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Fold/unfold alone has no way to reason about theories or do any quantifier instantiation beyond the instantiation of inductive definitions

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THEOREM 4.22. The Effectively Determined-Heap fragment of WSL admits a complete proof mechanism

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Thanks!