Summing Up Smart Transitions

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- Example: Transferring money

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transferFrom(a_1,a_2,n)
Post-conditions assert new-bal(a_1)=old-bal(a_1)-n and (i) new-bal(a_1)=old-bal(a_1)+n assert for each Address a≠a_1, a≠a_2: (ii) assert new-bal(a)=old-bal(a) assert new-sum()=old-sum() (iii)

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- Challenging for automated reasoning
- Abstract this *specific* setting away

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- Practice:
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  - Automated verification of 31 benchmarks

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- Theoretic results:
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  - Undecidable already with 3 `bal` functions

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- We use `Addr` elements to represent the time-steps of the machine.
- Different `bal` functions represent the content of the different registers.
- We use the sum as a way to count the number of time-steps.

Theoretical Gap

Our theoretical results have a gap — what about 2 `bal` functions?

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Soundness of the Encoding

Summation constant s satisfies the semantic property of SL

$$\sum_{a} \texttt{bal}(a) = s$$

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Two counting mechanisms.

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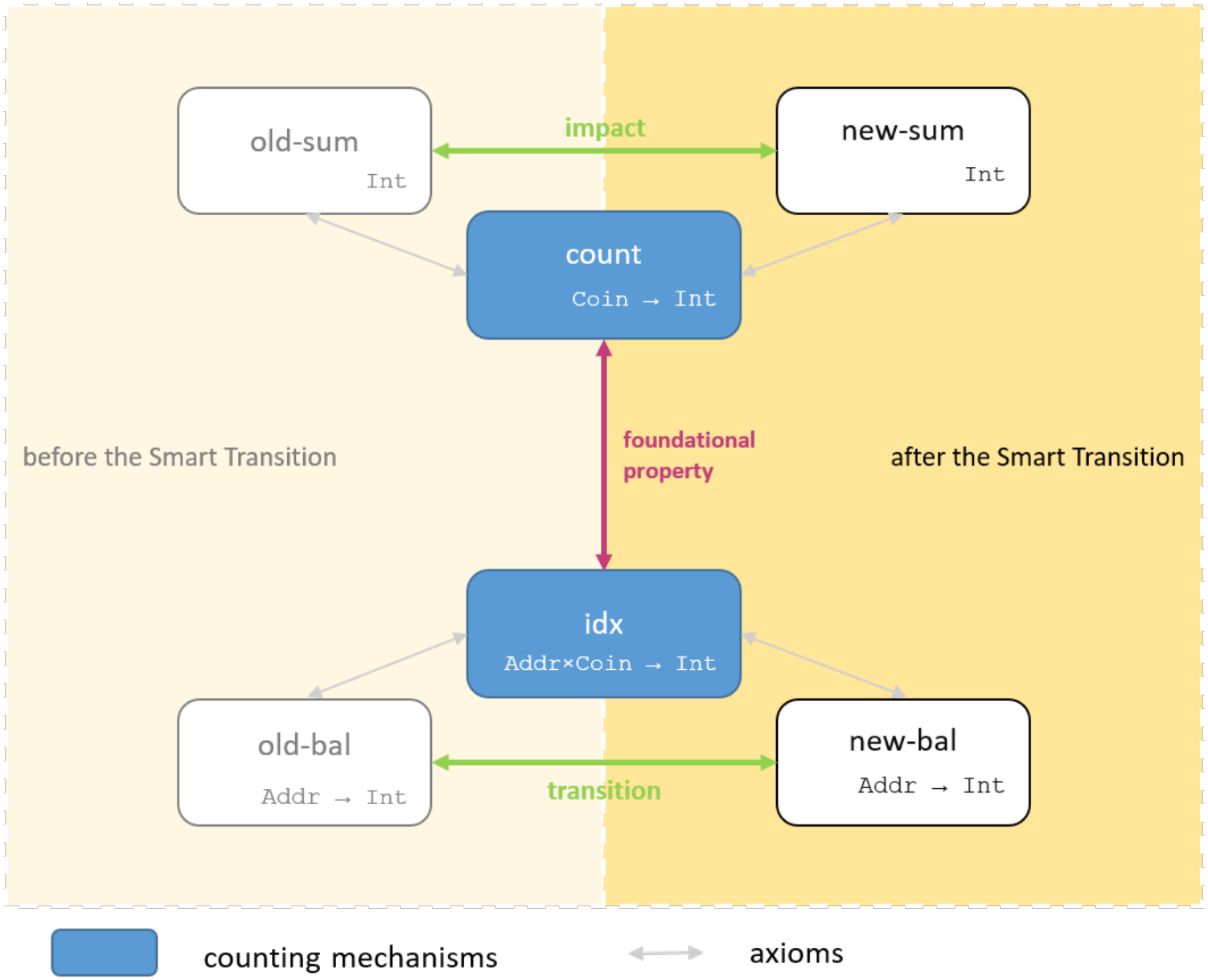
Related Addresses. A coin c is related to an address a iff $\texttt{idx}(a, c) \leq \texttt{bal}(a)$.

High-Level Overview



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- And other simple arithmetic problems
- For the $\text{transfer}_n, \text{mint}_n$ problems we provide a simplifying assumption

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- SMT-solvers Z3 and CVC4, first-order prover Vampire
- run on a standard machine
- time limit of 300s

Experiments

Version	Task							
	$\texttt{mint}_1$		$\texttt{transfer}_1$		$\texttt{mint}_n$		$\texttt{transfer}_n$	
`nat`	Z3:	0.02	Z3:	×	Z3:	×	Z3:	×
	CVC4:	×	CVC4:	×	CVC4:	×	CVC4:	×
	Vampire:	0.92	Vampire:	15.3	Vampire:	$23.2^{\dagger}$	Vampire:	$228^{\dagger}$
`int`	Z3:	0.02	Z3:	0.02	Z3:	0.03	Z3:	0.11
	CVC4:	0.03	CVC4:	0.07	CVC4:	0.05	CVC4:	0.35
	Vampire:	×	Vampire:	×	Vampire:	×	Vampire:	×
`id`	Z3:	×	Z3:	×	Z3:	×	Z3:	×
	CVC4:	×	CVC4:	×	CVC4:	×	CVC4:	×
	Vampire:	$7.36^{\ddagger}$	Vampire:	$17.1^{\ddagger}$	Vampire:	$23.5^{\ddagger}$	Vampire:	×

Table 1: Smart Transitions using different SL encodings.

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- For the arithmetic benchmarks, Z3 works very well

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  - multidimensional arrays