Problem #94

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Date: 1979

Summary: *Is* higher-order matching *decidable?*

Higher-order matching is the following problem:

Given a set of equations $s_i = t_i$ between typed lambda-terms where the t_i are ground, is there a substitution σ such that $\sigma s_i = t_i$ for all i. The order of the matching problem is the maximal height of function arrows in the types of the terms. Is higher-order matching decidable for arbitrary order? The problem has non-elementary complexity [Vor97].

The following results are known:

- First-order matching is, of course, decidable.
- Second-order matching is decidable [Hue76].
- Third-order matching is decidable [Dow93].
- Fourth-order matching is decidable [Pad96] and NEXPTIME-hard [Wie99]. The solutions can be described by a tree automaton [CJ97], which gives an 2-NEXPTIME upper bound.
- A restricted case of fifth-order matching has been shown decidable in [Sch97].
- Linear higher-order matching is decidable and NP-complete [dG00].

More on the complexity of higher-order matching can be found in [Wie99].

This problem is also listed as *Problem #21* in the TLCA list of open problems.

Remark

It has recently been announced [Sti06] that the higher-order matching problem is decidable. However, the proof method only applies to the "classical" case of the problem, that is when all types are built from a single atom. Therefore, the problem remains open for the generalized case of types built from an arbitrary number of type variables.

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