

Problem #93

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Date: 1996

Summary: Are the existential fragment or the positive fragment of the theory of one-step rewriting decidable?

For a given signature Σ and rewrite system R , the theory of one-step rewriting by R is the first order theory of the model comprising all Σ -ground-terms, and the binary predicate x rewrites to y in one rewrite step of R .

It is well-known that the full first-order theory is undecidable, even for strong restrictions on the class of rewrite systems (see Problem #51). Is the existential fragment of this theory (in other words: satisfiability of quantifier-free formulas) decidable? Is the positive fragment (arbitrary quantification, but no negation or implications) decidable?

It is known that the positive existential fragment is decidable [NPR97], and there are decidability results for the full existential fragment in case of restricted classes of rewrite systems [CSTT99, LR99].

Bibliography

- [CSTT99] Anne-Cécile Caron, Franck Seynhaeve, Sophie Tison, and Marc Tommasi. Deciding the satisfiability of quantifier free formulae on one-step rewriting. In Narendran and Rusinowitch [NR99], pages 103–117.
- [LR99] Sébastien Limet and Pierre Réty. A new result about the decidability of the existential one-step rewriting theory. In Narendran and Rusinowitch [NR99], pages 118–132.
- [NPR97] Joachim Niehren, Manfred Pinkal, and Peter Ruhrberg. On equality up-to constraints over finite trees, context unification and one-step rewriting. In William McClune, editor, *14th International Conference on Automated Deduction*, volume 1249 of *Lecture Notes in Artificial Intelligence*, pages 34–48, Townsville, Australia, July 1997. Springer-Verlag.
- [NR99] Paliath Narendran and Michael Rusinowitch, editors. *10th International Conference on Rewriting Techniques and Applications*, volume 1631 of *Lecture Notes in Computer Science*, Trento, Italy, July 1999. Springer-Verlag.
- [Tre96] Ralf Treinen. The first-order theory of one-step rewriting is undecidable. In Harald Ganzinger, editor, *7th International Conference on Rewriting Techniques and Applications*, volume 1103 of *Lecture Notes in Computer Science*, pages 276–286, New Brunswick, NJ, USA, July 1996. Springer-Verlag.

January 22, 2014