

Problem #87 (Solved !)

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Summary: Is it decidable whether a single term rewrite rule can be proved terminating by a monotonic ordering that is total on ground terms?

Termination of string-rewriting systems is known to be undecidable [HL78]. Termination of a single term-rewriting rule was proved undecidable in [Dau92, Les94]. It is also undecidable whether there exists a simplification ordering that proves termination of a single term rewriting rule [MG95] (cf. [JK84]). Is it decidable whether a single term rewrite rule can be proved terminating by a monotonic ordering that is total on ground terms? (With more rules it is not [Zan94].)

Remark

A negative solution has been given in [GMOZ97]. More about the history of this problem in the context of the question of one-rule termination can be found in [Der05].

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