Problem #84

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Summary: Is unification of patterns modulo any set of variable-preserving equations decidable?

Unification of patterns (à la [Mil91]) modulo associativity and commutativity has been shown decidable [BC97], repairing the incomplete solution in [QW94]. Does it extend to equational theories whose axioms have the same set of variables on left and right hand side?

Comment sent by Evelyne Contejean

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In his conference paper, Qian claimed that he has solved the problem of unifying patterns a la Miller modulo AC, but in fact he never succeeded to prove the completeness of his algorithm. Actually his algorithm is not complete, since he uses a first-order unification algorithm for pure AC-patterns as a black box. The problem was solved last year by Boudet and Contejean [BC97]: the case of pure AC-patterns requieres is handled in the same spirit as the first order case, by counting things, but technically this is not exactly identical. In [BC97], the proof of completeness of the algorithm is given. I must admit that [BC97] takes advantage of the paper of Qian, in particular, the remark that the equations of the form

$$\lambda x_1 ... x_n F(x_1, ..., x_n) = \lambda x_1 ... x_n F(x_{\pi(1)}, ..., x_{\pi(n)})$$

have an infinite set of solutions $\{\sigma_1, \sigma_2, ...\}$ such that σ_{i+1} is strictly more general than σ_i. This leads to the notion of constrained solution of a unification problem, and every unification problem of patterns with AC symbols admits a finite complete set of constrained unifiers, and the algorithm proposed in [BC97] computes such a set.

Bibliography

[BC97] Alexandre Boudet and Evelyne Contejean. AC-unification of higher-order patterns. In Gert Smolka, editor, *Principles and Practice of Constraint Programming*, volume 1330 of *Lecture Notes in Computer Science*, pages 267–281, Linz, Austria, October 1997. Springer-Verlag.

[Mil91] Dale Miller. A logic programming language with lambda-abstraction, function variables, and simple unification. In P. Schroeder-Heister, editor, *Extensions of Logic Programming*, volume 690 of *Lecture Notes in Computer Science*. Springer-Verlag, 1991.

[QW94] Zheniu Qian and Kang Wang. Modular AC-unification of higher-order patterns. In Jean-Pierre Jouannaud, editor, *First International Conference on Constraints in Computational Logics*, volume 845 of *Lecture Notes in Computer Science*, pages 105–120, München, Germany, September 1994. Springer-Verlag.

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