Problem #68 (Solved !)

Originator: Hubert Comon
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Summary: *Is satisfiability of* set constraints with projection and boolean operators *decidable?*

Consider the existential fragment of the theory defined by a binary predicate symbol $\subseteq$, a finite set of function symbols $f_1, \ldots, f_n$, the function symbols $\cap, \cup, \neg$, and the projection symbols $f_{i,j}^{-1}$ for $j \leq arity(f_i)$. Variables are interpreted as subsets of the Herbrand Universe. With the obvious interpretation of these symbols, is satisfiability of such formulas decidable? Special cases have been solved in [HJ90, AW92, BGW93, GTT93a].

Remark

This has been solved positively by [CP94b].

Partial solutions have been given by [GTT93b][CP94a][AKW93].

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