Problem #62 (Solved !)

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Summary: Is the union of two left-linear, confluent combinatory reduction systems over the same alphabet, where the rules of the first system do not overlap the rules of the second, confluent?

Let R and S be two left-linear, confluent combinatory reduction systems with the same alphabet. Suppose the rules of R do not overlap the rules of S. Is $R \cup S$ confluent? This is true for the restricted case when R is a term-rewriting system (an easy generalization of a result by F. Müller [Mül92]), or if neither system has critical pairs. (The restriction to the same alphabet is essential, since confluence is in general not preserved under the addition of function symbols, not even for left-linear systems.)

Remark

The answer is yes (Thm. 3.1 of [vOvR94]).

Bibliography

[Mül92]   Fritz Müller. Confluence of the lambda calculus with left-linear algebraic rewriting. *Information Processing Letters*, 41:293–299, April 1992.

[vOvR94] Vincent van Oostrom and Femke van Raamsdonk. Weak orthogonality implies confluence: the higher-order case. In A. Nerode and Y. V. Matiyasevich, editors, *Third International Symposium on the Logical Foundations of Computer Science*, volume 813 of *Lecture Notes in Computer Science*, pages 379–392, St. Petersburg, Russia, July 1994. Springer-Verlag.

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