

## Problem #61 (Solved !)

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*Summary: Are weakly orthogonal higher-order rewrite systems confluent?*

For higher-order rewrite formats as given by combinatory reduction systems [Klo80] and higher-order rewrite systems [Nip91, Tak93], confluence has been proved in the restricted case of orthogonal systems. Can confluence be extended to such systems when they are weakly orthogonal (all critical pairs are trivial)? When critical pairs arise only at the root, confluence is known to hold.

### Remark

Weakly orthogonal higher-order rewriting systems are confluent. This has been shown both via the Tait-Martin-Löf method and via finite developments [vOvR94].

The details and further extensions similar to Huet's parallel closure condition can be found in [Oos94, Oos97, Raa96].

# Bibliography

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