Problem #51 (Solved !)

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> Summary: Is the first order theory of one-step rewriting decidable?

For an arbitrary finite term rewriting system R, is the first order theory of one-step rewriting (\rightarrow_R) decidable? Decidability would imply the decidability of the first-order theory of encompassment (that is, being an instance of a subterm) [CCD93], as well as several known decidability results in rewriting. (It is well known that the theory of \rightarrow_R^* is in general undecidable.)

Remark

This has been answered negatively in [Tre96, Tre98]. Sharper undecidability results have been obtained for the following subclasses of rewrite systems:

- linear, shallow, $\exists^*\forall^*$ -fragment ([STT97], [STTT01]);
- linear, terminating, $\exists^*\forall^*\exists^*$ -fragment ([Vor97]), $\exists^*\forall^*$ -fragment ([Mar97]).
- right-ground, terminating, $\exists^* \forall^*$ -fragment ([Mar97]).

Decidbability results have been obtained for

- the positive existential theory ([NPR97])
- unary signatures ([Jac96])
- left-linear right-ground systems ([Tis90])

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January 22, 2014

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