Problem #5

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Summary: Does surjective pairing conservatively extend $\lambda\beta\eta$-conversion?

Do the surjective pairing axioms

$$\begin{aligned} D_1(Dxy) &= x \\ D_2(Dxy) &= y \\ D(D_1x)(D_2x) &= x \end{aligned}$$

conservatively extend $\lambda\beta\eta$-conversion on pure untyped lambda terms? More generally, is surjective pairing *always* conservative, or do there exist lambda theories, or extensions of Combinatory Logic for that matter, for which conservative extension by surjective pairing fails? (Surjective pairing is conservative over the pure $\lambda\beta$-calculus; see [dV89]). Of course, there are lots of other $\lambda\beta$, indeed $\lambda\beta\eta$, theories where conservative extension holds, simply because the theory consists of the valid equations in some λ model in which surjective pairing functions exist, e.g., D_∞.

Comment sent by Kristian Støvring

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The problem has been solved with a positive answer [Stø05, Stø06]. The generalization to arbitrary lambda theories remains open.

Bibliography

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[Stø06] Kristian Støvring. Extending the extensional lambda calculus with surjective pairing is conservative. *Logical Methods in Computer Science*, 2(2:1):1–14, 2006.

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