Problem #48 (Solved !)

Originator: H.-C. Kong
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Summary: Is embedding a well-quasi-ordering on strings?

Consider the following relation on strings over an infinite set $\mathcal{X}$ of variables: $x_1x_2\cdots x_m \hookrightarrow y_1y_2\cdots y_n$ if there exists a renaming $\rho : \mathcal{X} \to \mathcal{X}$ such that $x_i\rho = y_{j_i}$ for $1 \leq j_1 < j_2 < \cdots < j_m \leq n$. Is this "embedding" relation $\hookrightarrow$ a well-quasi-ordering (that is, must every infinite sequence of strings contain two strings, such that the first embeds in the second)?

Remark

The answer is "yes". (Map each variable to the position of its leftmost occurrence and use the fact that strings of natural numbers are well-quasi-ordered by the embedding extension of $\leq$ to strings.)

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