Problem #25 (Solved !)

Originator: Ralf Treinen [Tre90]
Date: April 1991

Summary: Is the Σ_2-fragment of the first-order theory of ground terms modulo AC decidable?

Consider a finite set of function symbols containing at least one AC (associative-commutative) function symbol. Let T be the corresponding set of terms (modulo the AC properties). It is known from [Tre92] that the first-order theory (Σ_3 fragment) of T is undecidable when F contains at least a non-constant symbol (besides the AC symbol). When F only contains an AC symbol and constants, the theory reduces to Presburger's arithmetic and is hence decidable. On the other hand the Σ_1 fragment of T is always decidable [Com93]. The decidability of the Σ_2 fragment of the theory of T remains open.

Remark

Even more, the solvability of the following important particular case is open: given $t, t_1, \ldots, t_n \in T(F, X)$, is there an instance of t which is not an instance of $t_1, \ldots, t_n$ modulo the AC axioms? This is known as *complement problems* modulo AC.

Several special cases have been solved [Fer93][LM93], and in unpublished work in progress.

The undecidability of the Σ_2-fragment of the first-order theory of ground terms modulo AC has been shown by [Mar99].

Bibliography

[Com93] Hubert Comon. Complete axiomatizations of some quotient term algebras. *Theoretical Computer Science*, 118(2), September 1993.

[Fer93] Maribel Fernández. AC-complement problems: Validity and negation elimination. In Claude Kirchner, editor, *5th International Conference on Rewriting Techniques and Applications*, volume 690 of *Lecture Notes in Computer Science*, pages 358–373, Montreal, Canada, June 1993. Springer-Verlag.

[LM93] D. Lugiez and J.-L. Moysset. Complement problems and tree automata in AC-like theories. In *Proceedings of the Symposium on Theoretical Aspects of Computer Science (Würzburg, Germany)*, Lecture Notes in Computer Science, Berlin, 1993. Springer-Verlag.

[Mar99] Jerzy Marcinkowski. Undecidability of the $\exists^*\forall^*$ part of the theory of ground term algebra modulo an AC symbol. In Paliath Narendran and Michael Rusinowitch, editors, *10th International Conference on Rewriting Techniques and Applications*, volume 1631 of *Lecture Notes in Computer Science*, pages 92–102, Trento, Italy, July 1999. Springer-Verlag.

[Tre90] Ralf Treinen. A new method for undecidability proofs of first order theories. In K. V. Nori and C. E. Veni Madhavan, editors, *Proceedings of the Tenth Conference on Foundations of Software Technology and Theoretical Computer Science*, volume 472 of *Lecture Notes in Computer Science*, pages 48–62. Springer-Verlag, 1990.

[Tre92] Ralf Treinen. A new method for undecidability proofs of first order theories. *Journal of Symbolic Computation*, 14(5):437–458, November 1992.

January 22, 2014