Prophets and Secretaries with Overbooking

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The prophet and secretary problems demonstrate online scenarios involving the optimal stopping theory. In a typical prophet or secretary problem, selection decisions are assumed to be immediate and irrevocable. However, many online settings accommodate some degree of revocability. To study such scenarios, we introduce the ℓ*-out-of-*k setting, where the decision maker can select up to k elements immediately and irrevocably, but her performance is measured by the top ℓ elements in the selected set. Equivalently, the decision makes can hold up to ℓ elements at any given point in time, but can make up to $k - \ell$ returns as new elements arrive. We give upper and lower bounds on the competitive ratio of ℓ*-out-of-*k prophet and secretary scenarios.

For ℓ*-out-of-*k prophet scenarios we provide a single-sample algorithm with competitive ratio $1-\ell\cdot e^{-\Theta\left(\frac{(k-\ell)^2}{k}\right)}$. The algorithm is a single-threshold algorithm, which sets a threshold that equals the $(\frac{\ell+k}{2})^{th}$ highest sample, and accepts all values exceeding this threshold, up to reaching capacity k. On the other hand, we show that this result is tight if the number of possible returns is linear in ℓ (i.e., $k - \ell = \Theta(\ell)$). In particular, we show that no single-sample algorithm obtains a competitive ratio better than $1 - \frac{2^{-(2k+1)}}{k+1}$.

We also present a deterministic single-threshold algorithm for the 1*-out-of-*k *prophet* setting which obtains a competitive ratio of $1 - \frac{3}{2} \cdot e^{-k/6}$, knowing only the distribution of the maximum value. This result improves the result of [Assaf & Samuel-Cahn, J. of App. Prob., 2000].

Furthermore, we show that no ℓ*-out-of-*k *prophet* algorithm, even one that has full information on the distributions of values from the outset, can achieve a better competitive ratio than $1 - \frac{1}{(2k+2)!}$.

For ℓ*-out-of-*k *secretary* scenarios, we provide an algorithm with a competitive ratio $1 - \ell e^{-\frac{k-4\ell}{2+2\ln \ell}} - e^{-k/6}$. The algorithm divides the values into $\ell + 1$ segments, numbered from 0 to ℓ. In the j-th segment the algorithm accepts the i^{th} element if it belongs to the j highest values seen so far, and the capacity k is not exhausted. On the negative side, we show that no ℓ*-out-of-*k *secretary* algorithm achieves a better competitive ratio than $1 - \frac{1}{e^k} + \frac{2}{3n}$.

Beyond the contribution to online algorithms and optimal stopping theory, our results have implications to mechanism design. In particular, we use our prophet algorithms to derive *overbooking* mechanisms with good welfare and revenue guarantees; these are mechanisms that sell more items than the seller's capacity, then allocate to the agents with the highest values among the selected agents.

Our results are summarized in Tables 1 and 2 below.

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CCS Concepts: • **Theory of computation → Online algorithms**; **Algorithmic mechanism design**;

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	Lower bound	Upper bound
Single-sample algorithm	$1 - \ell \cdot e^{-\Theta\left(\frac{(k-\ell)^2}{k}\right)}$	$1 - \frac{2^{-(2k+1)}}{k+1}$
D_{max} algorithm	$1 - \frac{3}{2} \cdot e^{-k/6}$	$1 - \frac{1}{(2k+2)!}$

Table 1. *ℓ-out-of-k prophet*

	Lower bound	Upper bound
Secretary	$1 - \ell e^{-\frac{k-4\ell}{2+2\ln \ell}} - e^{-k/6}$	$1 - \frac{1}{e^k} + \frac{2}{3n}$

Table 2. *ℓ-out-of-k secretary*

Additional Key Words and Phrases: prophet inequality, secretary problem, online algorithms, mechanism design, welfare approximation