Compilation

0368-3133

Lecture 5:

Syntax Analysis:

Bottom-Up Parsing

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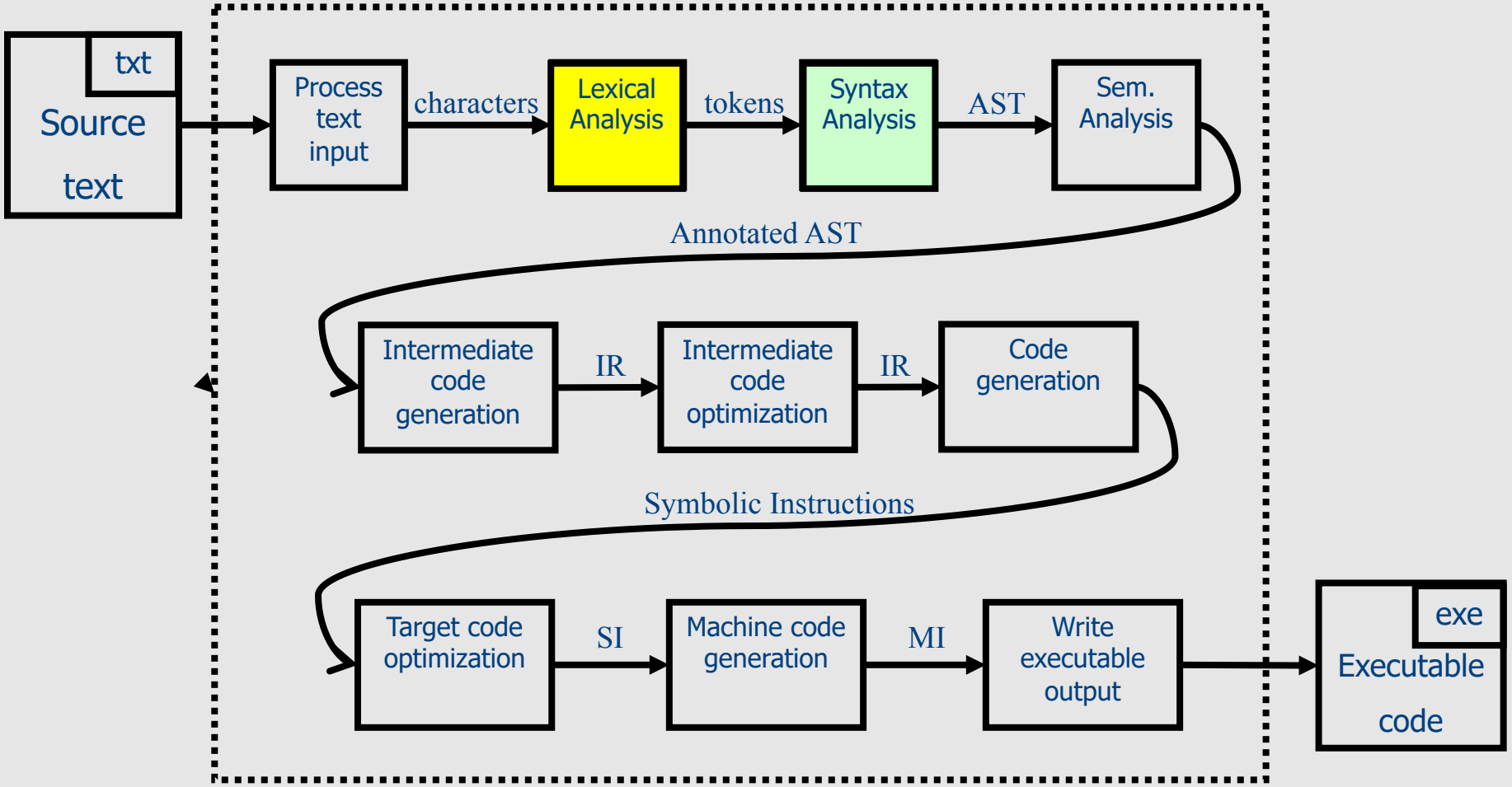
The Real Anatomy of a Compiler



Broad kinds of parsers

- Parsers for arbitrary grammars
  - Earley's method, CYK method
  - Usually, not used in practice (though might change)
- Top-Down parsers
  - Construct parse tree in a top-down matter
  - Find the leftmost derivation
- Bottom-Up parsers
  - Construct parse tree in a bottom-up manner
  - Find the rightmost derivation in a reverse order

Intuition: Top-down parsing

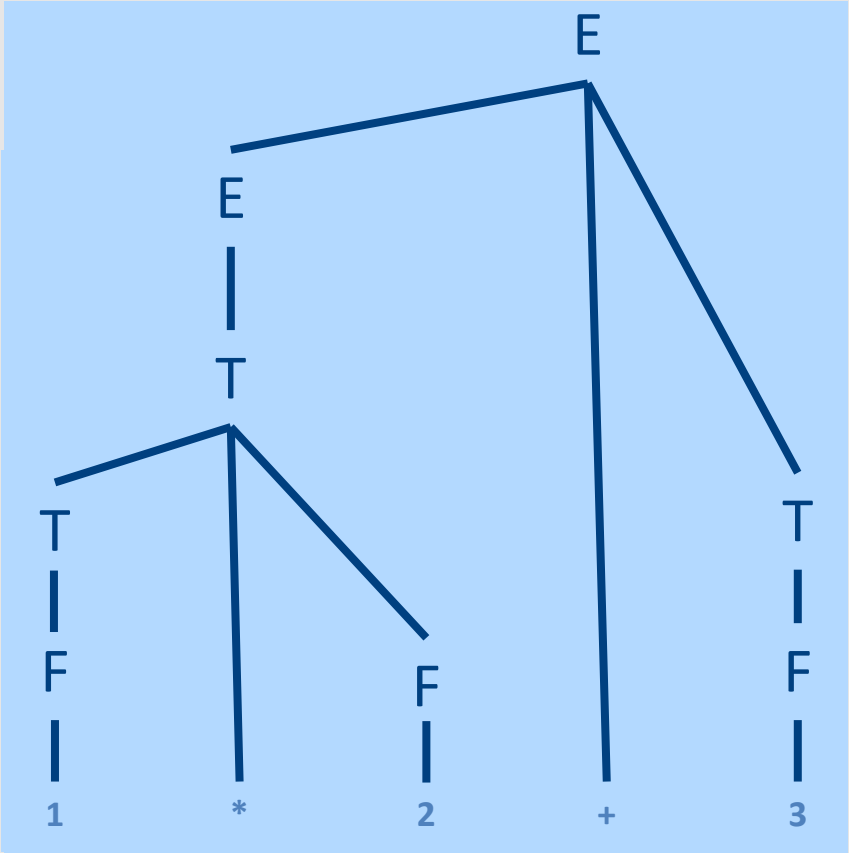
- Begin with start symbol
- Guess the productions
- Check if parse tree yields user's program

Bottom-Up Parsing

- Goal: Build a parse tree
  - Report error if input is not a legal program
- How:
  - Read input left-to-right
  - Construct a subtree for the first left-most tree node whose children have been constructed

Bottom-up parsing

$E \rightarrow E + T$
$E \rightarrow T$
$T \rightarrow T * F$
$T \rightarrow F$
$F \rightarrow id$
$F \rightarrow num$
$F \rightarrow (E)$



Bottom-up parsing: LR(k) Grammars

- A grammar is in the class LR(K) when it can be derived via:
  - Bottom-up derivation
  - Scanning the input from left to right (L)
  - Producing the rightmost derivation (R)
    - In reverse order
  - With lookahead of k tokens (k)
- A language is said to be LR(k) if it has an LR(k) grammar
- The simplest case is LR(0), which we will discuss

Terminology: Reductions & Handles

- The opposite of derivation is called *reduction*
  - Let $A \rightarrow \alpha$ be a production rule
  - Derivation: $\beta A \mu \rightarrow \beta \alpha \mu$
  - Reduction: $\beta \alpha \mu \rightarrow \beta A \mu$

- A *handle* is the reduced substring
  - α is the handles for $\beta \alpha \mu$

Use Shift & Reduce

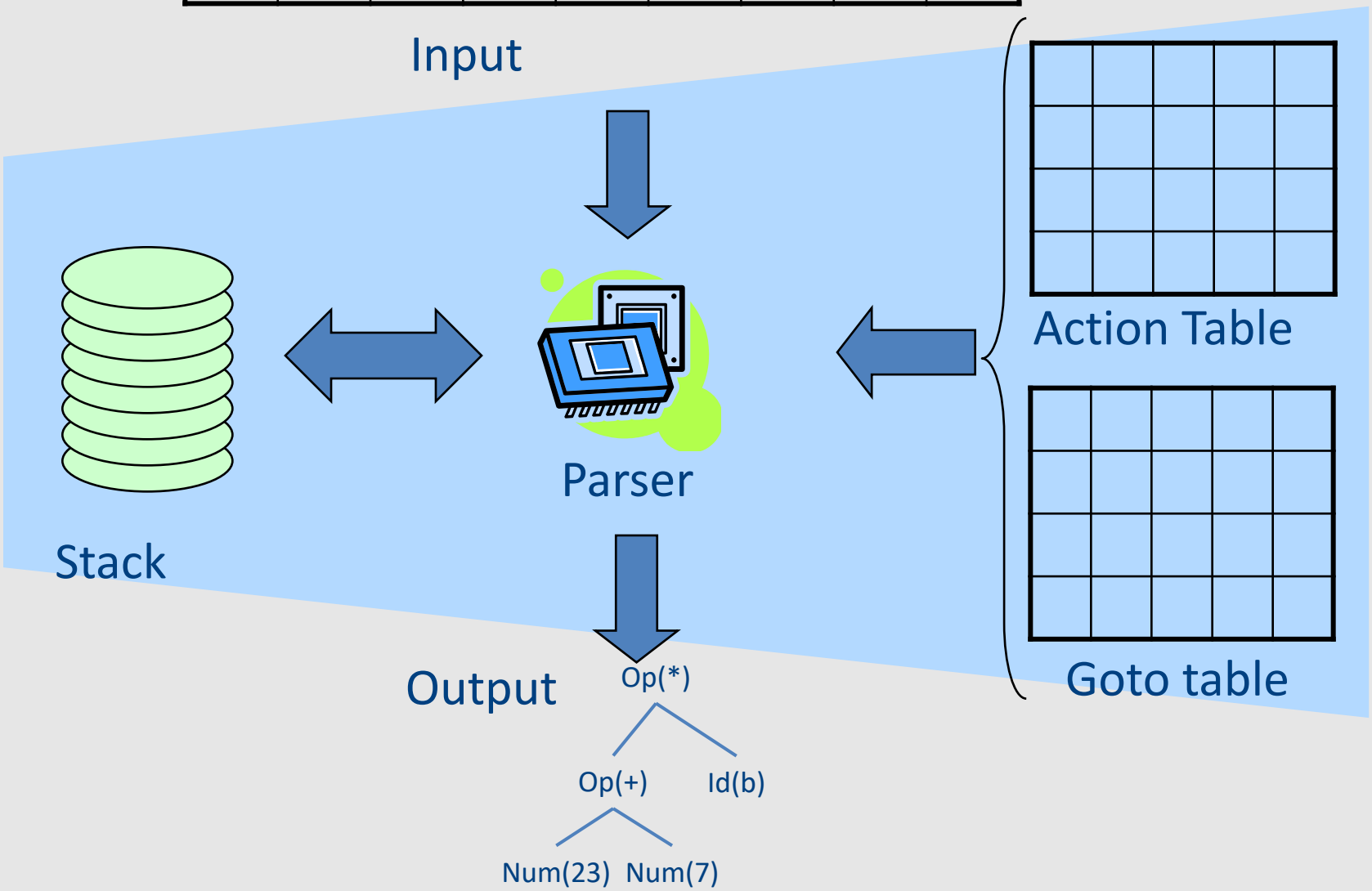
In each stage, we

- shift a symbol from the input to the stack, or
- reduce according to one of the rules.

How does the parser know what to do?

token stream

(	(	23	+	7	)	*	x	)
LP	LP	Num	OP	Num	RP	OP	Id	RP



How does the parser know what to do?

- A state will keep the info gathered on handle(s)
  - A state in the "control" of the PDA
  - Also (part of) the stack alpha bet

Set of LR(0) items

- A table will tell it "what to do" based on current state and next token
  - The transition function of the PDA

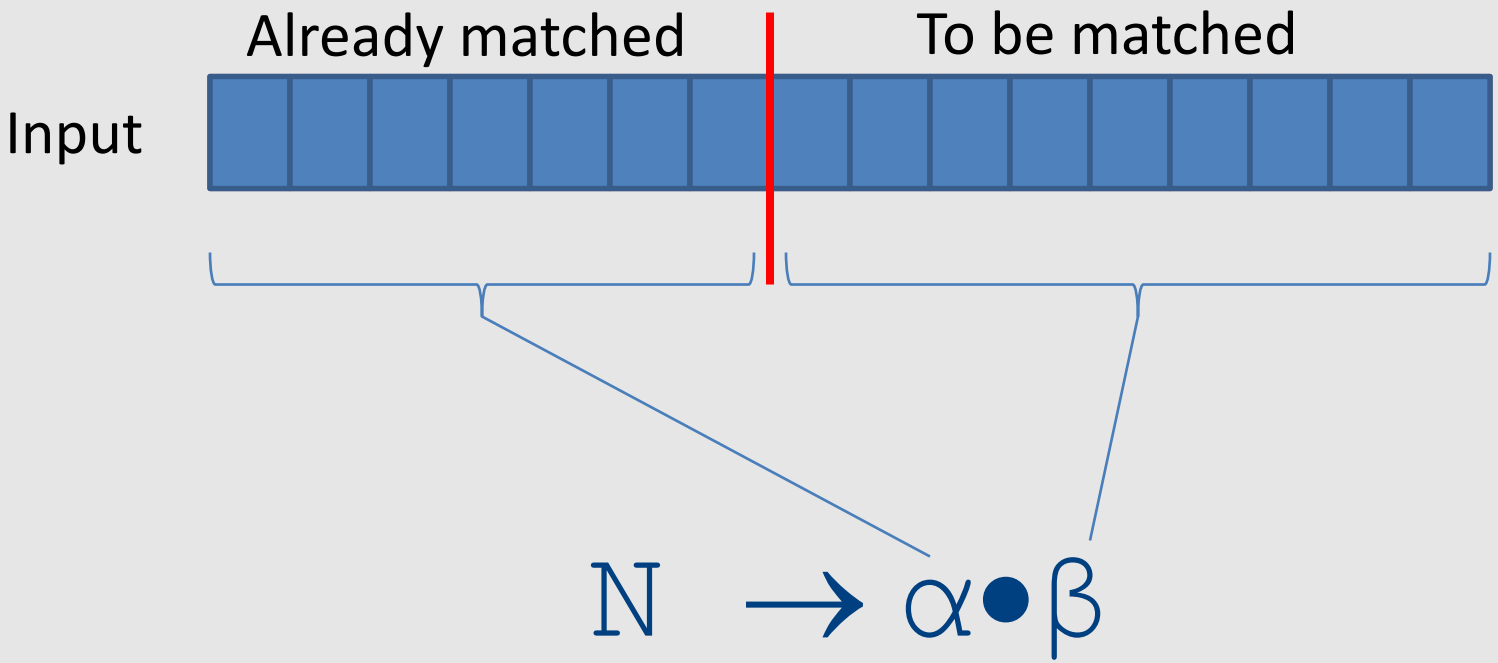
- A stack will records the "nesting level"
  - Stack contains a sequence of prefixes of handles

Important Bottom-Up LR-Parsers

- LR(0) – simplest, explains basic ideas
- SLR(1) – simple, explains look ahead
- LR(1) – complicated, very powerful, expensive
- LALR(1) – complicated, powerful enough, used by automatic tools

LR(0) Parsing

LR item



Hypothesis about αβ being a possible handle:
so far we've matched α, expecting to see β

Example: LR(0) Items

- All items can be obtained by placing a dot at every position for every production:

Grammar

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

LR(0) items

1: S → •E$
2: S → E • $
3: S → E $ •
4: E → • T
5: E → T •
6: E → • E + T
7: E → E • + T
8: E → E + • T
9: E → E + T •
10: T → • i
11: T → i •
12: T → • (E)
13: T → (• E)
14: T → (E •)
15: T → (E) •

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Grammar

(1) S → E $
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LR(0) items

1: S → •E$
2: S → E • $
3: S → E $ •
4: E → • T
5: E → T •
6: E → • E + T
7: E → E • + T
8: E → E + • T
9: E → E + T •
10: T → • id
11: T → id •
12: T → • (E)
13: T → (• E)
14: T → (E •)
15: T → (E) •

- Before • = reduced
  - matched prefix
- After • = may be reduced
  - May be matched by suffix

LR(0) items

$N \rightarrow \alpha \bullet \beta$ Shift Item

$N \rightarrow \alpha \beta \bullet$ Reduce Item

States are sets of items

LR(0) Items

$E \rightarrow E * B \mid E + B \mid B$
$B \rightarrow 0 \mid 1$

- A derivation rule with a location marker (●) is called LR(0) item

PDA States

$E \rightarrow E * B \mid E + B \mid B$
$B \rightarrow 0 \mid 1$

- A PDA state is a set of LR(0) items. E.g.,

  $q_{13} = \{ E \rightarrow E \bullet * B ,\ E \rightarrow E \bullet + B,\ B \rightarrow 1 \bullet \}$

- Intuitively, if we matched 1,

  Then the state will remember the 3 possible alternatives rules and where we are in each of them

(1) $E \rightarrow E \bullet * B$ (2) $E \rightarrow E \bullet + B$ (3) $B \rightarrow 1 \bullet$

LR(0) Shift/Reduce Items

$N \rightarrow \alpha \bullet \beta$ Shift Item

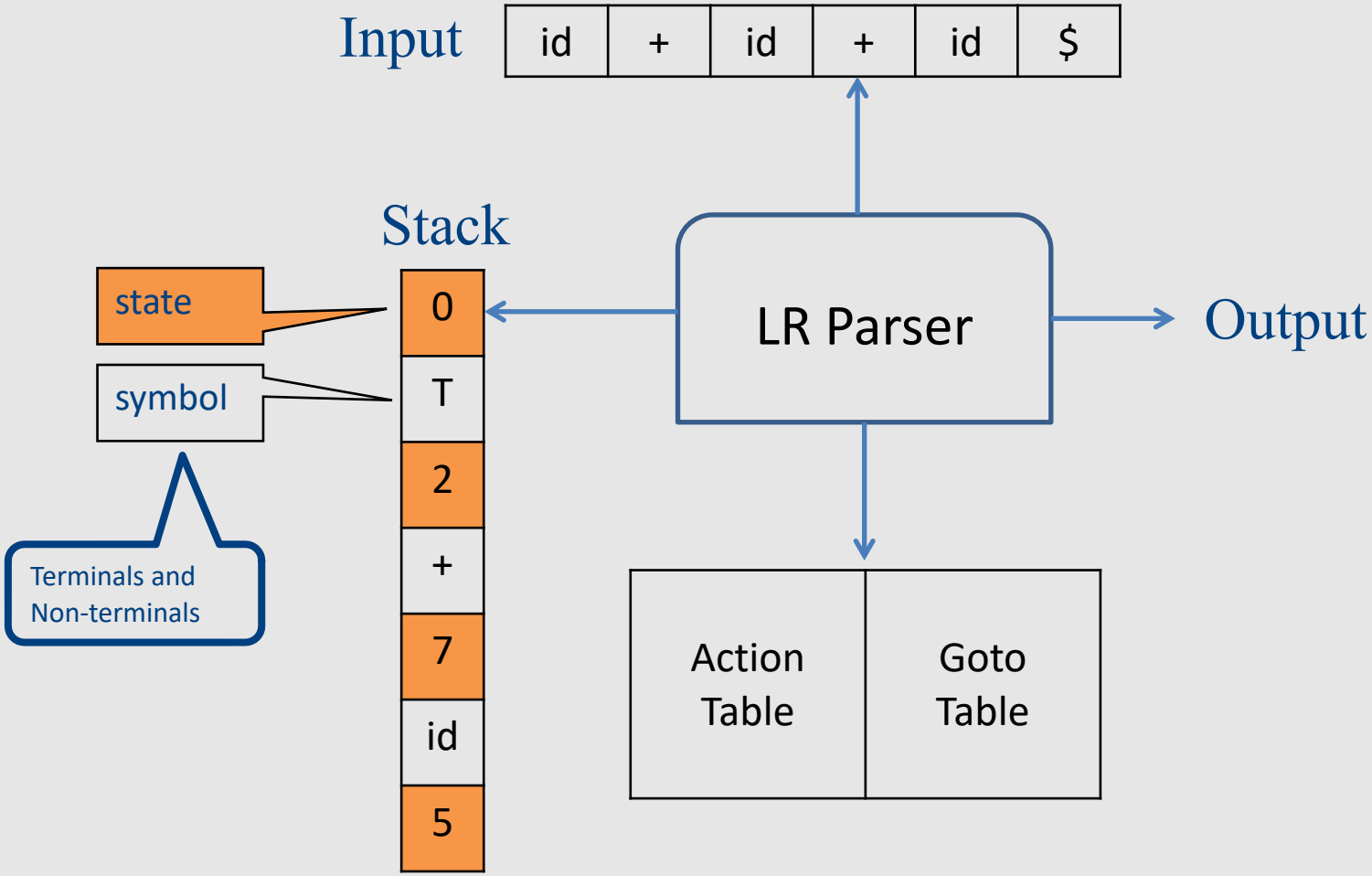
$N \rightarrow \alpha \beta \bullet$ Reduce Item

Intuition

- Read input tokens left-to-right and remember them in the stack
- When a right hand side of a rule is found, remove it from the stack and replace it with the non-terminal it derives

- Remembering token is called shift
  - Each shift moves to a state that remembers what we've seen so far
- Replacing RHS with LHS is called reduce
  - Each reduce goes to a state that determines the context of the derivation

Model of an LR parser



LR parser stack

- Sequence made of state, symbol pairs
- For instance a possible stack for the grammar

  $S \rightarrow E\ \$$

  $E \rightarrow T$

  $E \rightarrow E + T$

  $T \rightarrow id$

  $T \rightarrow (\ E\ )$

  could be: 0 T 2 + 7 id 5

Stack grows this way

Form of LR parsing table

state	terminals	non-terminals
0	Shift/Reduce actions	Goto part
1	acc	
.		gm
.	rk	
.	sn	
	error	

shift state *n*

reduce by rule *k*

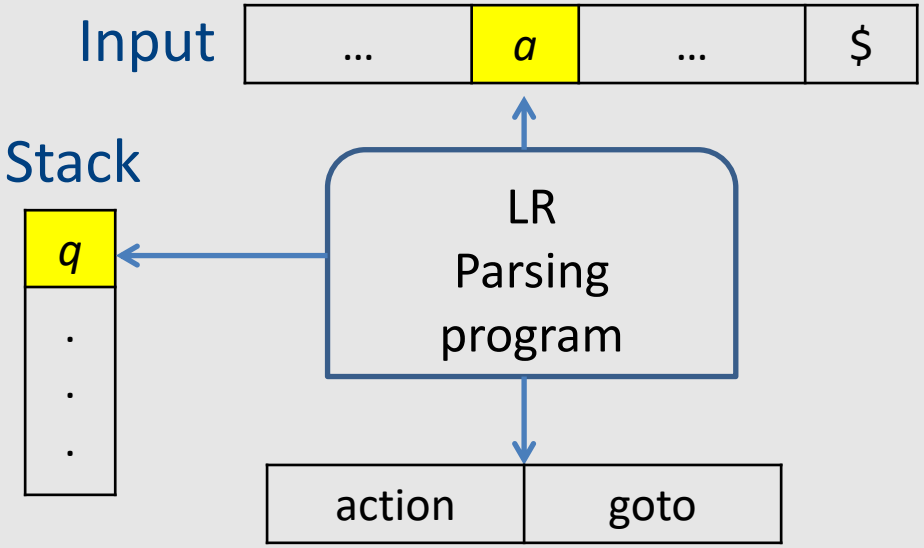
goto state *m*

accept

LR parser table example

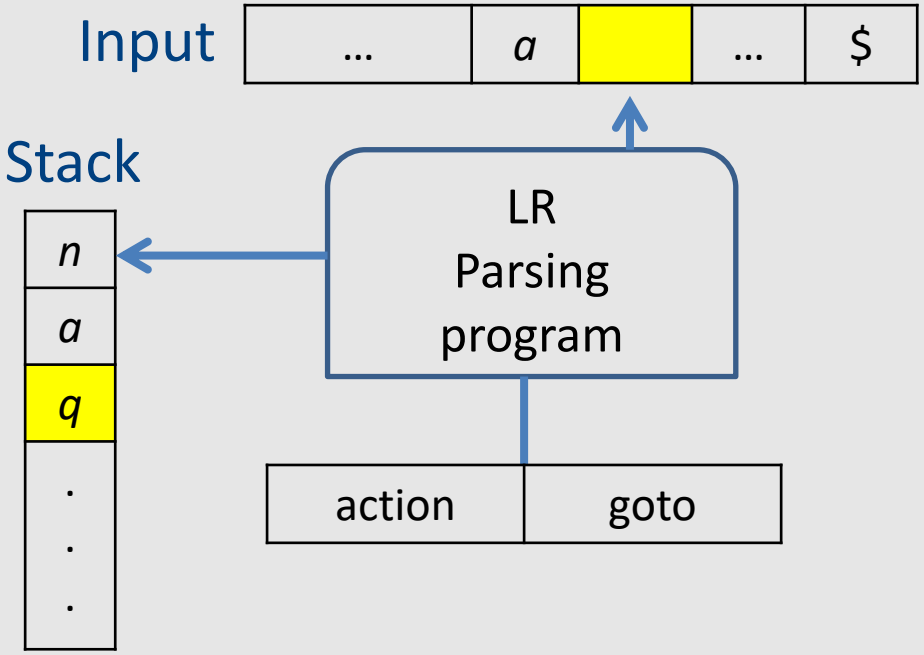
STATE	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

Shift move



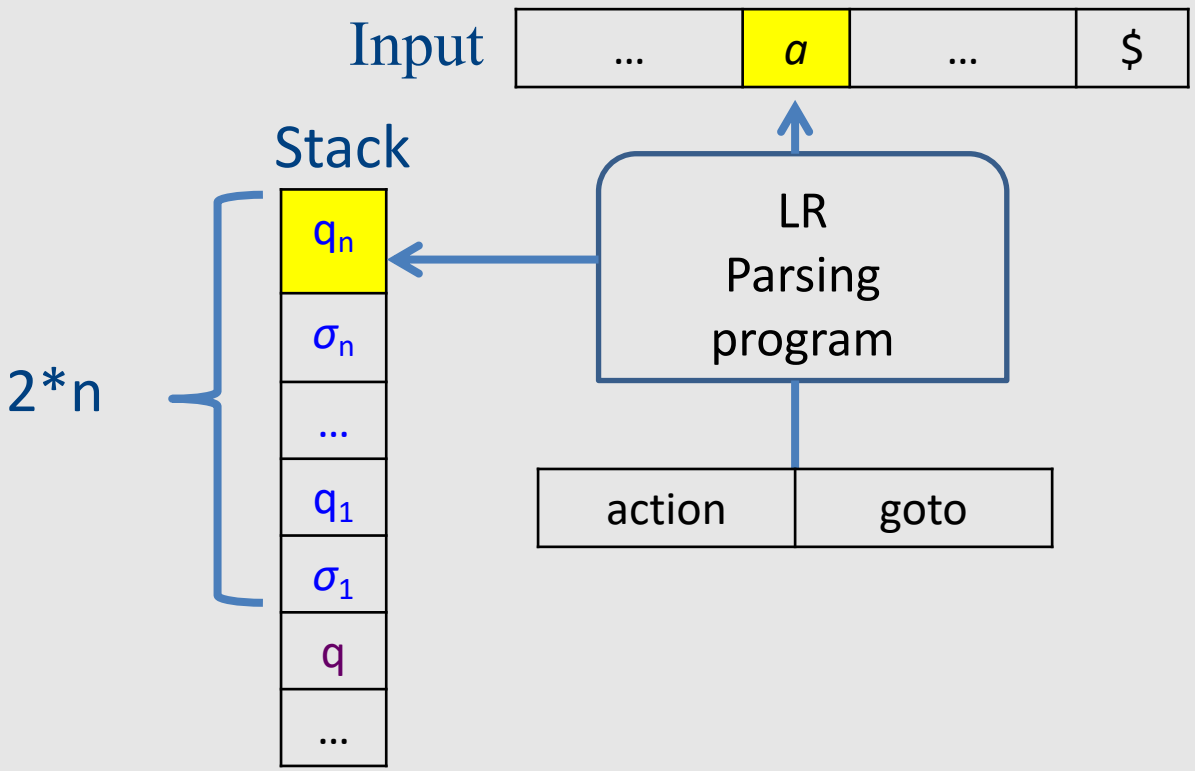
- action[*q*, *a*] = s*n*

Result of shift



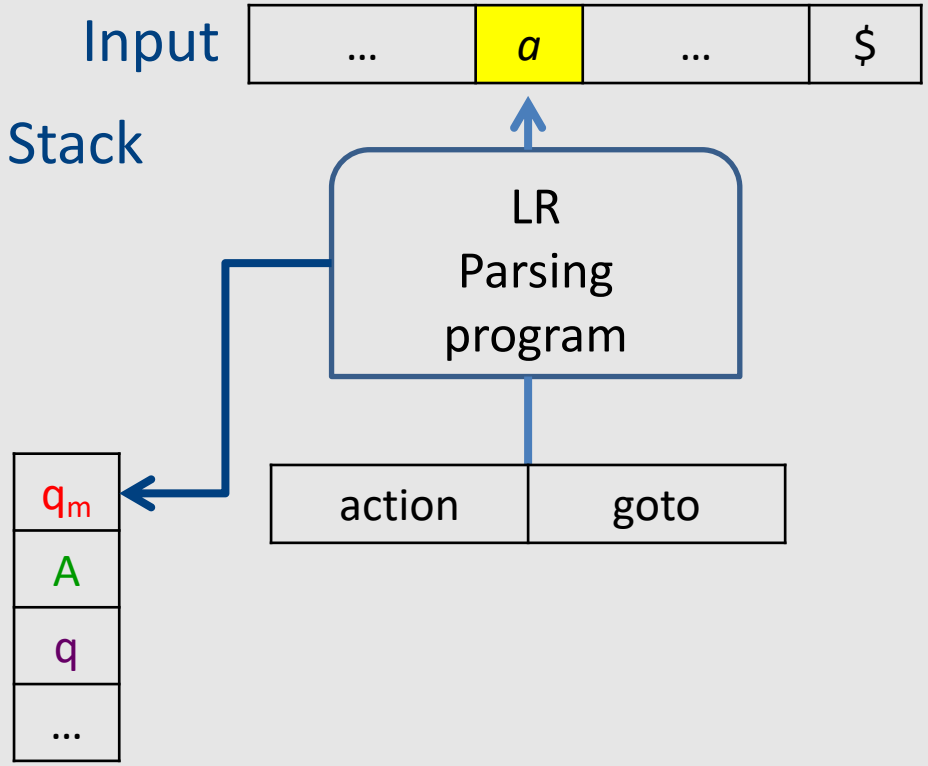
- action[q, a] = sn

Reduce move



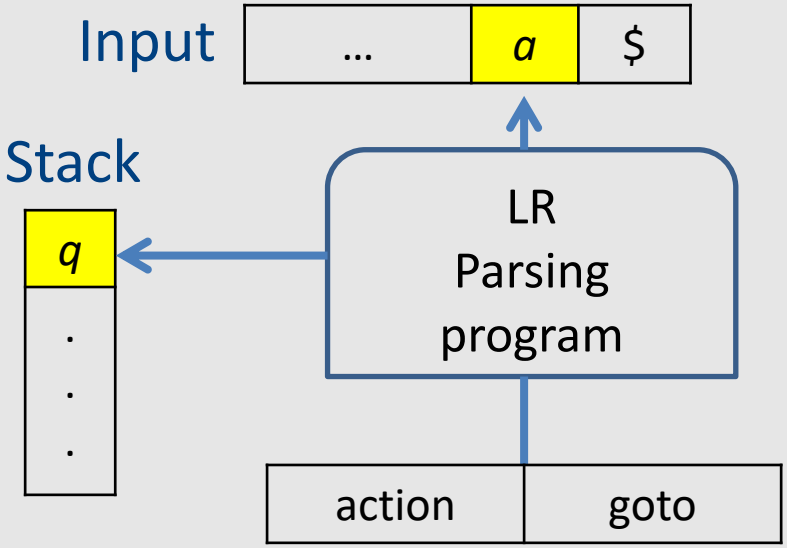
- action[q_n, a] = r*k*
- Production: (k) A → $\sigma_1 \ldots \sigma_n$
- Top of stack looks like $q_1\sigma_1 \ldots q_n\sigma_n$ for some $q_1 \ldots q_n$
- goto[q, A] = q_m

Result of reduce move



- action[q_n, a] = r*k*
- Production: (k) A → $\sigma_1 \ldots \sigma_n$
- Top of stack looks like $q_1\sigma_1 \ldots q_n\sigma_n$ for some $q_1 \ldots q_n$
- goto[q, A] = q_m

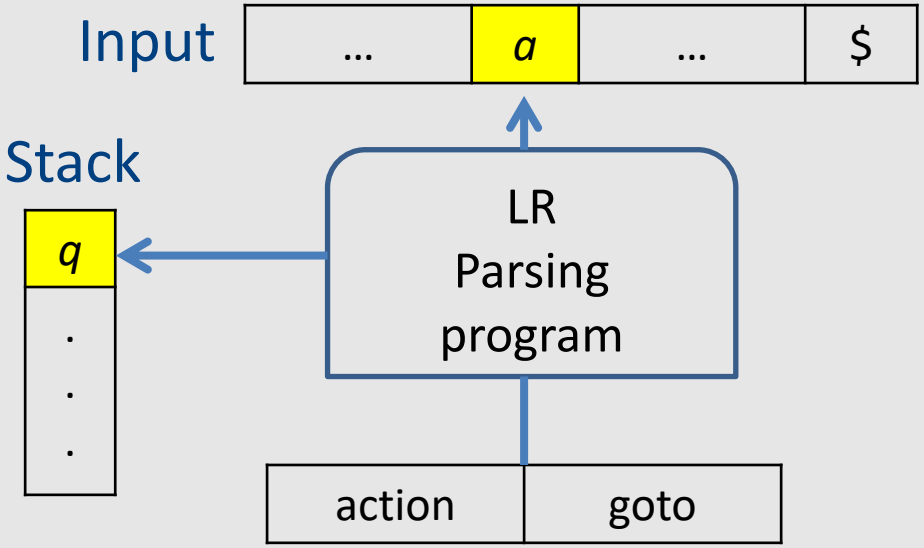
Accept move



If action[q, a] = accept

parsing completed

Error move



If action[q, a] = error (usually empty)

parsing discovered a syntactic error

Example

Z → E **$**

E → T | E **+** T

T → **i** | **(** E **)**

Example: parsing with LR items

```
Z -> E $
E -> T | E + T
T -> i | ( E )
```

	i		+		i		$

```
Z -> •E $
E -> •T
E -> •E + T
T -> •i
T -> •( E )
```

Why do we need these additional LR items?
Where do they come from?
What do they mean?

ε-closure

- Given a set S of LR(0) items

- If P –> α•Nβ is in state S
- then for each rule N –> X in the grammar state S must also contain N –> •X

ε-closure ({Z –> •E $}) = { Z –> •E $,
E –> •T,
E –> •E + T,
T –> •i ,
T –> •(E) }

```
Z -> E $
E -> T | E + T
T -> i | ( E )
```

Example: parsing with LR items

	i		+		i		$

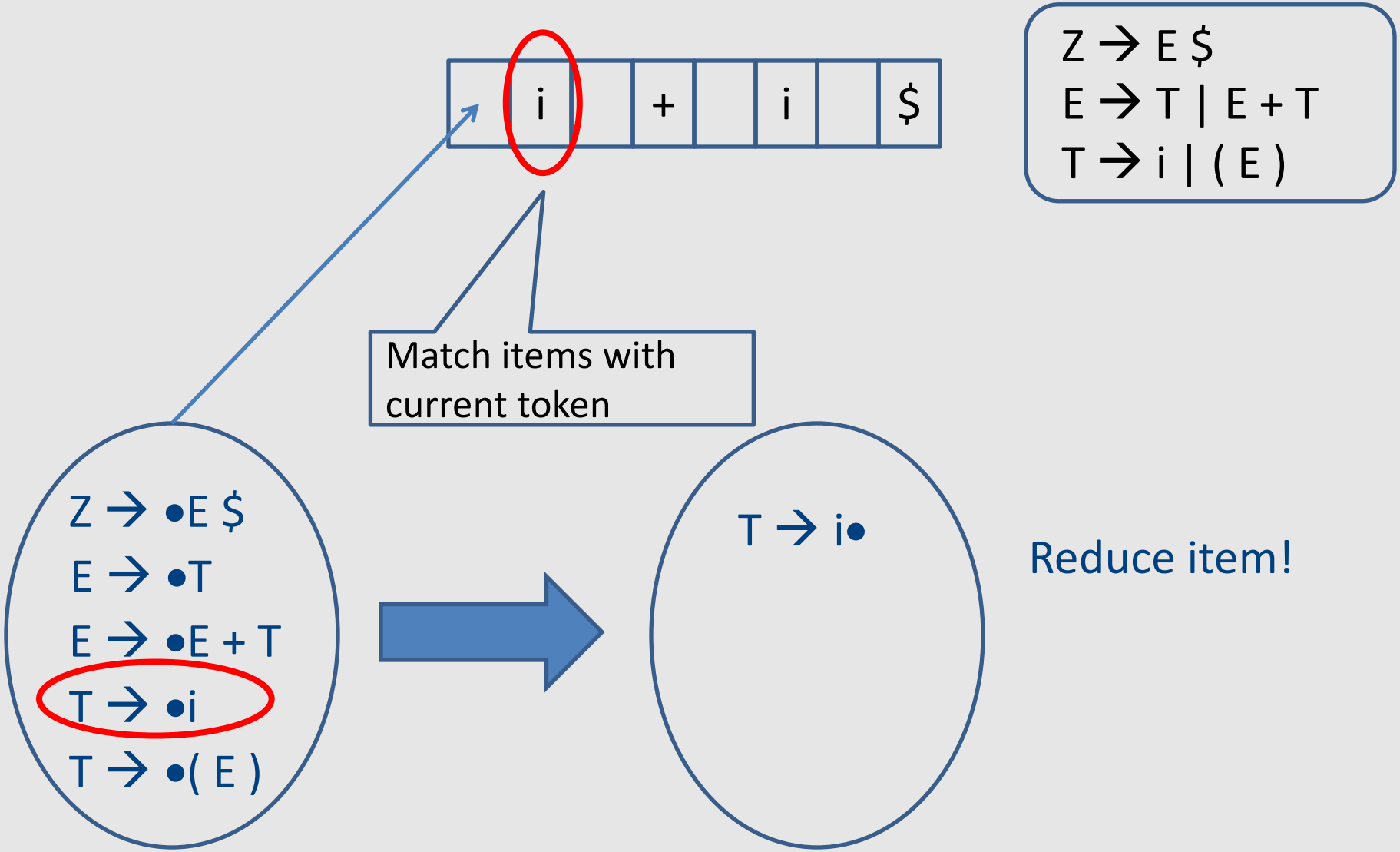
Remember position from which we're trying to reduce

Items denote possible future handles

Z → •E $
E → •T
E → •E + T
T → •i
T → •(E)

Z → E $
E → T | E + T
T → i | (E)

Example: parsing with LR items



Example: parsing with LR items

	T		+		i		$

i

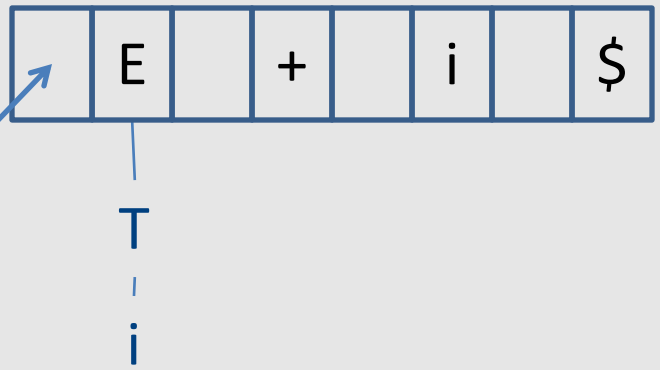
Z → E $
E → T | E + T
T → i | (E)

Z ⟶ •E $
E ⟶ •T
E ⟶ •E + T
T ⟶ •i
T ⟶ •(E)

E ⟶ T•

Reduce item!

Example: parsing with LR items



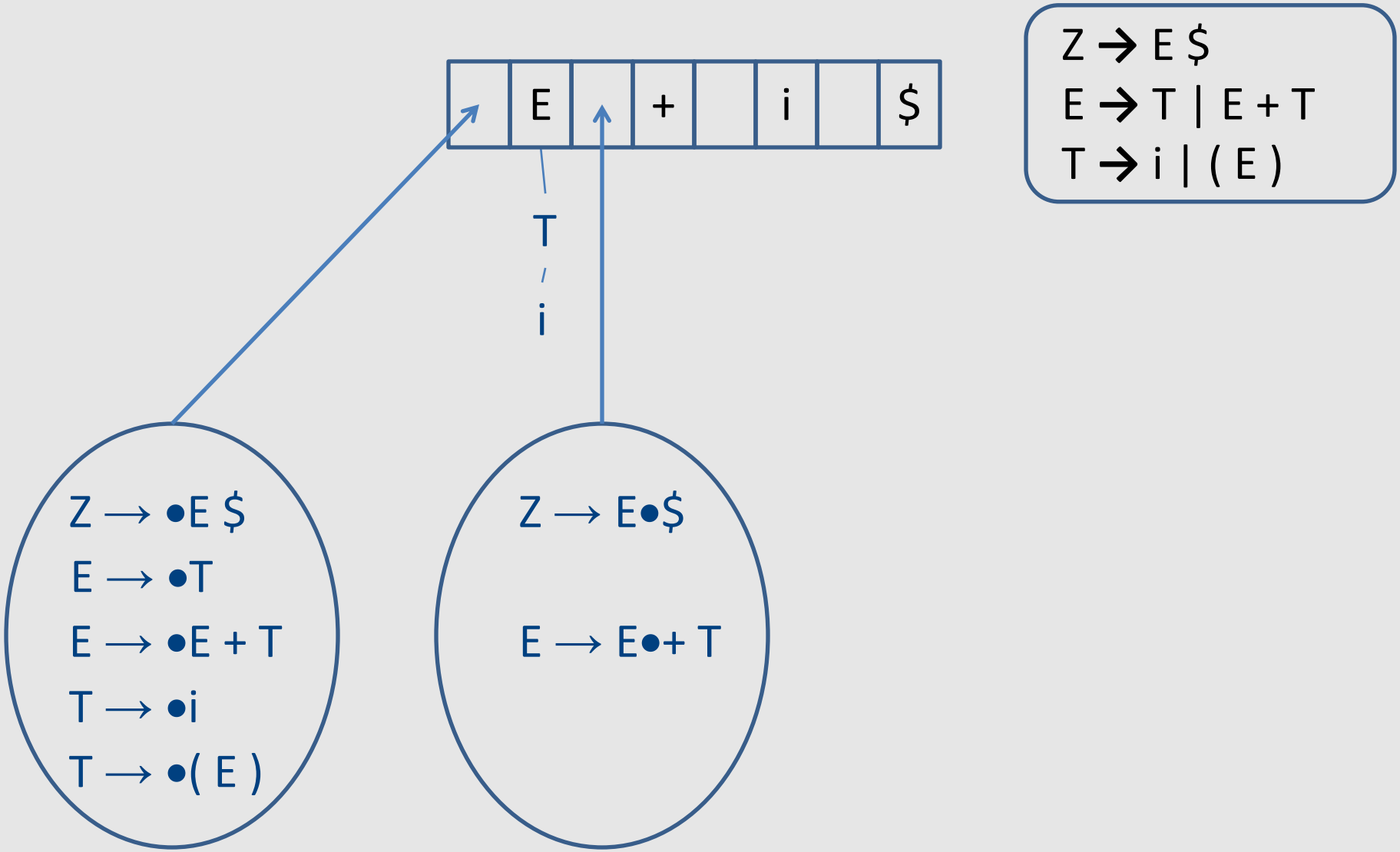
Z → E $
E → T | E + T
T → i | (E)

Z → •E $
E → •T
E → •E + T
T → •i
T → •(E)

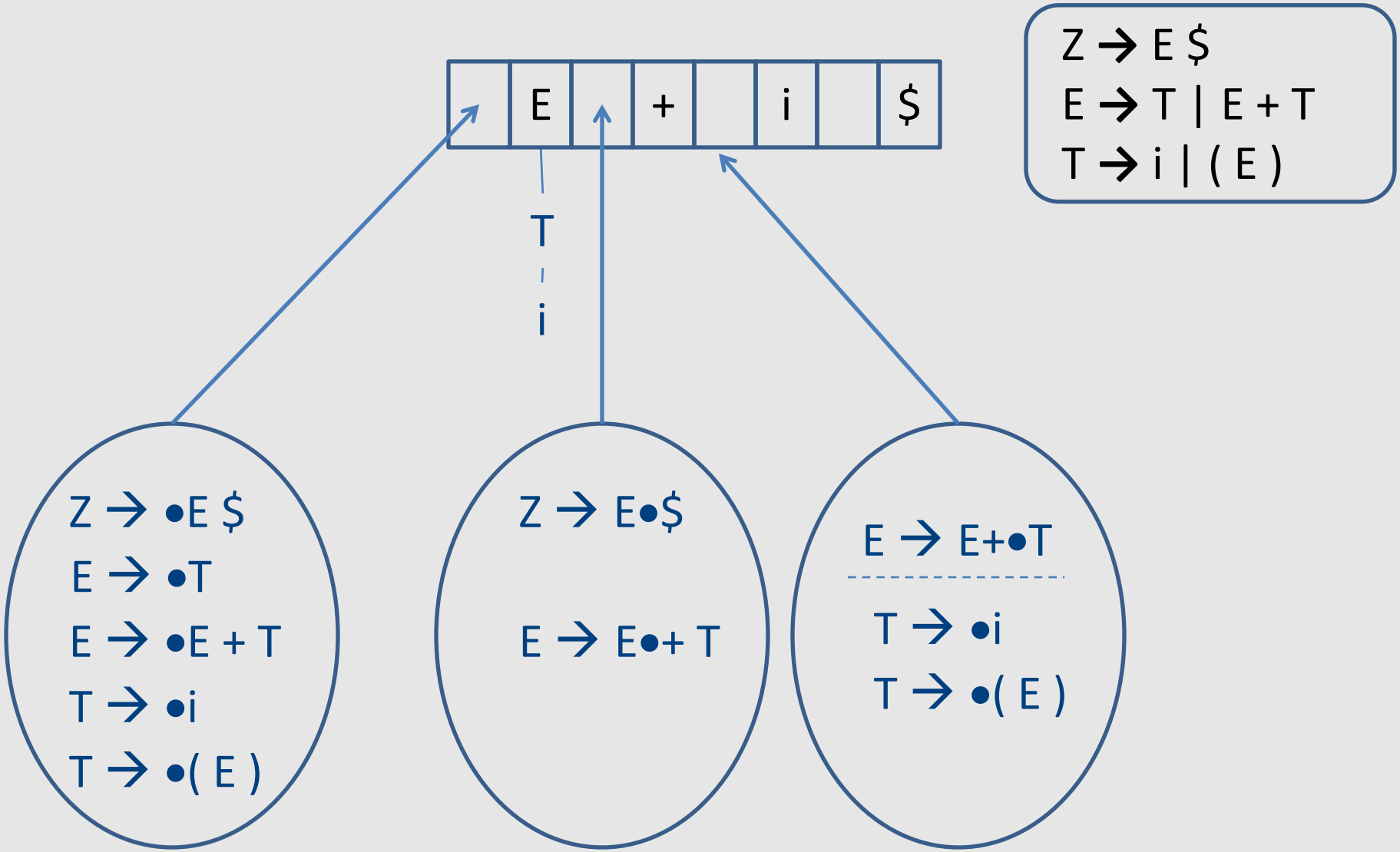
E → T•

Reduce item!

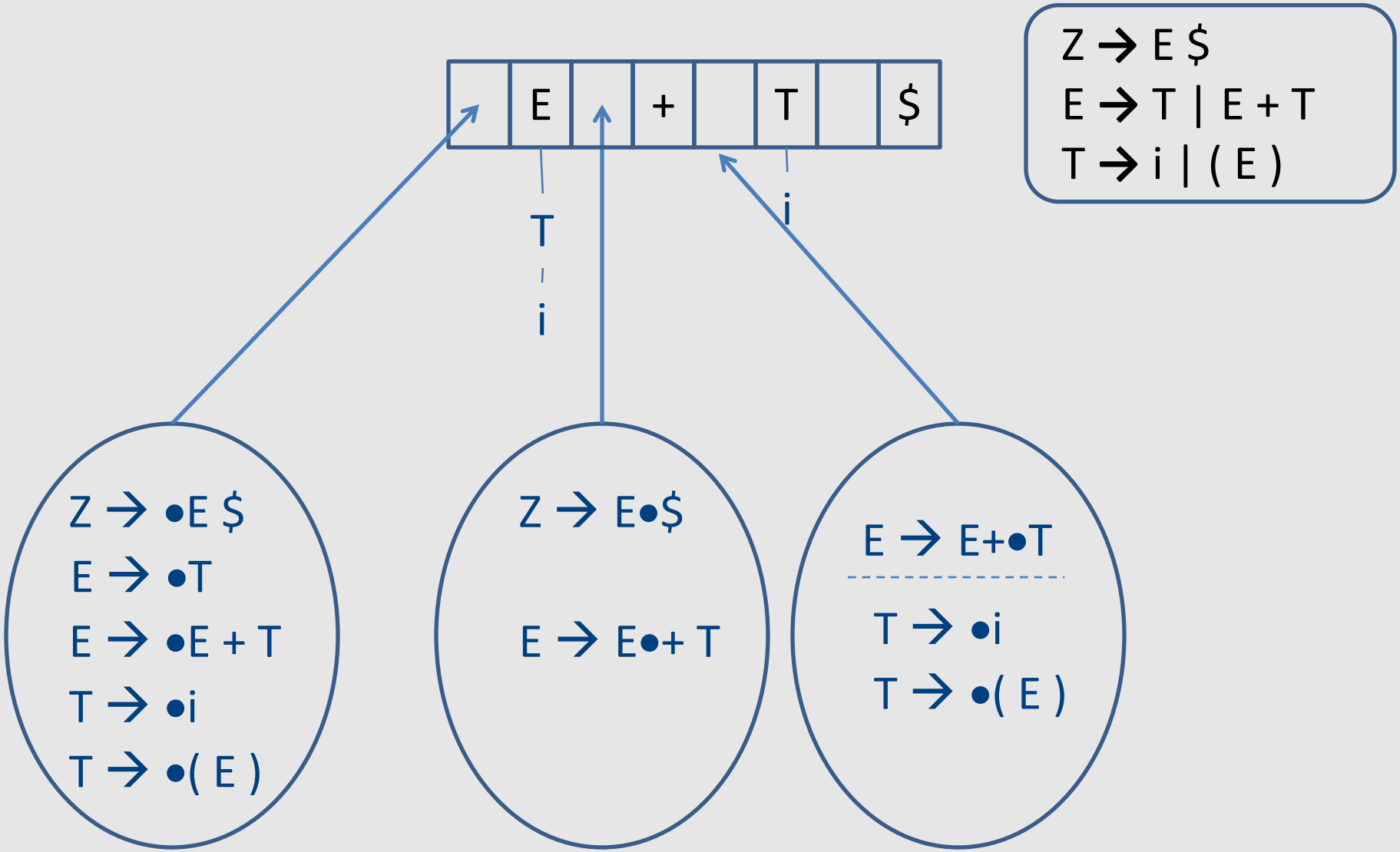
Example: parsing with LR items



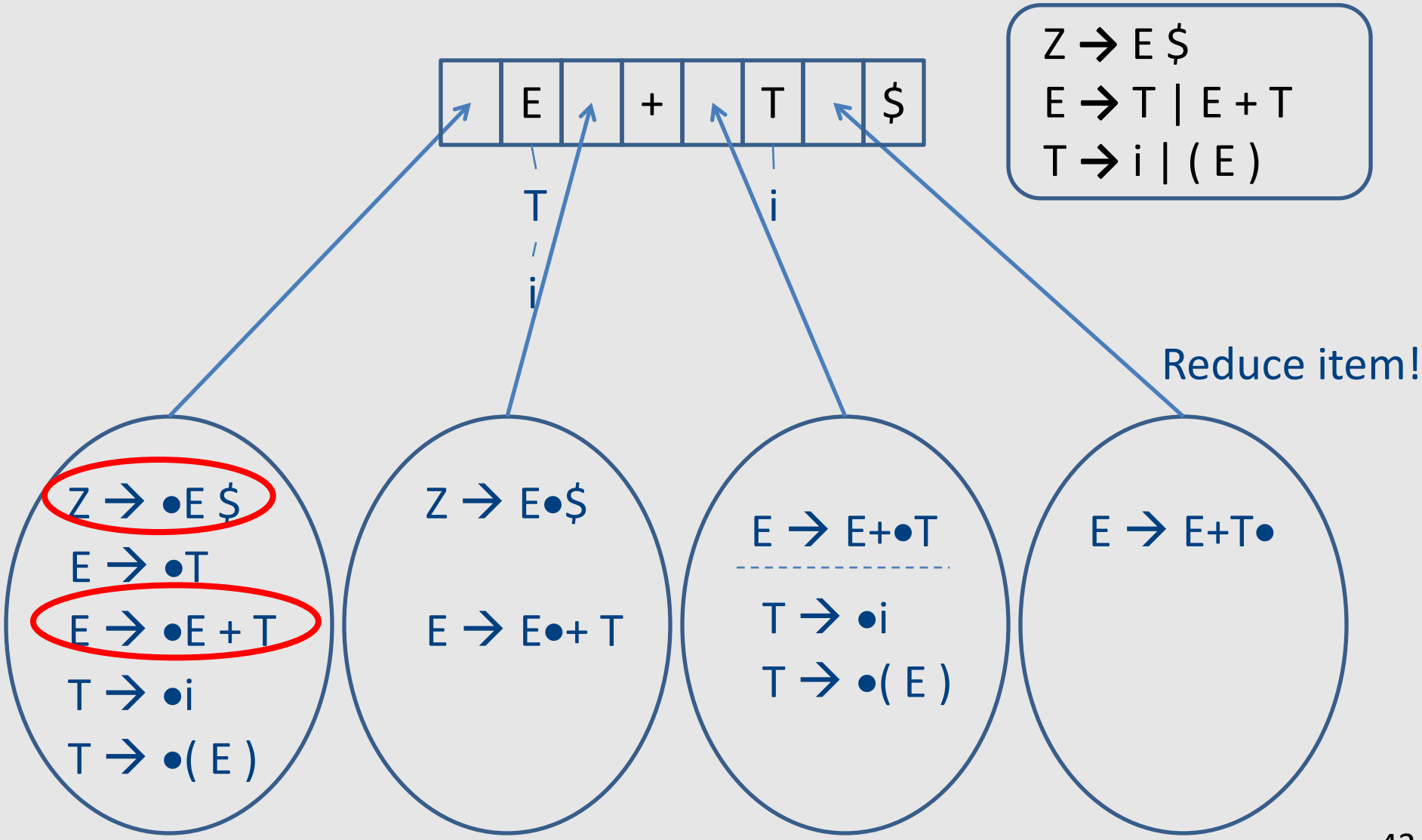
Example: parsing with LR items



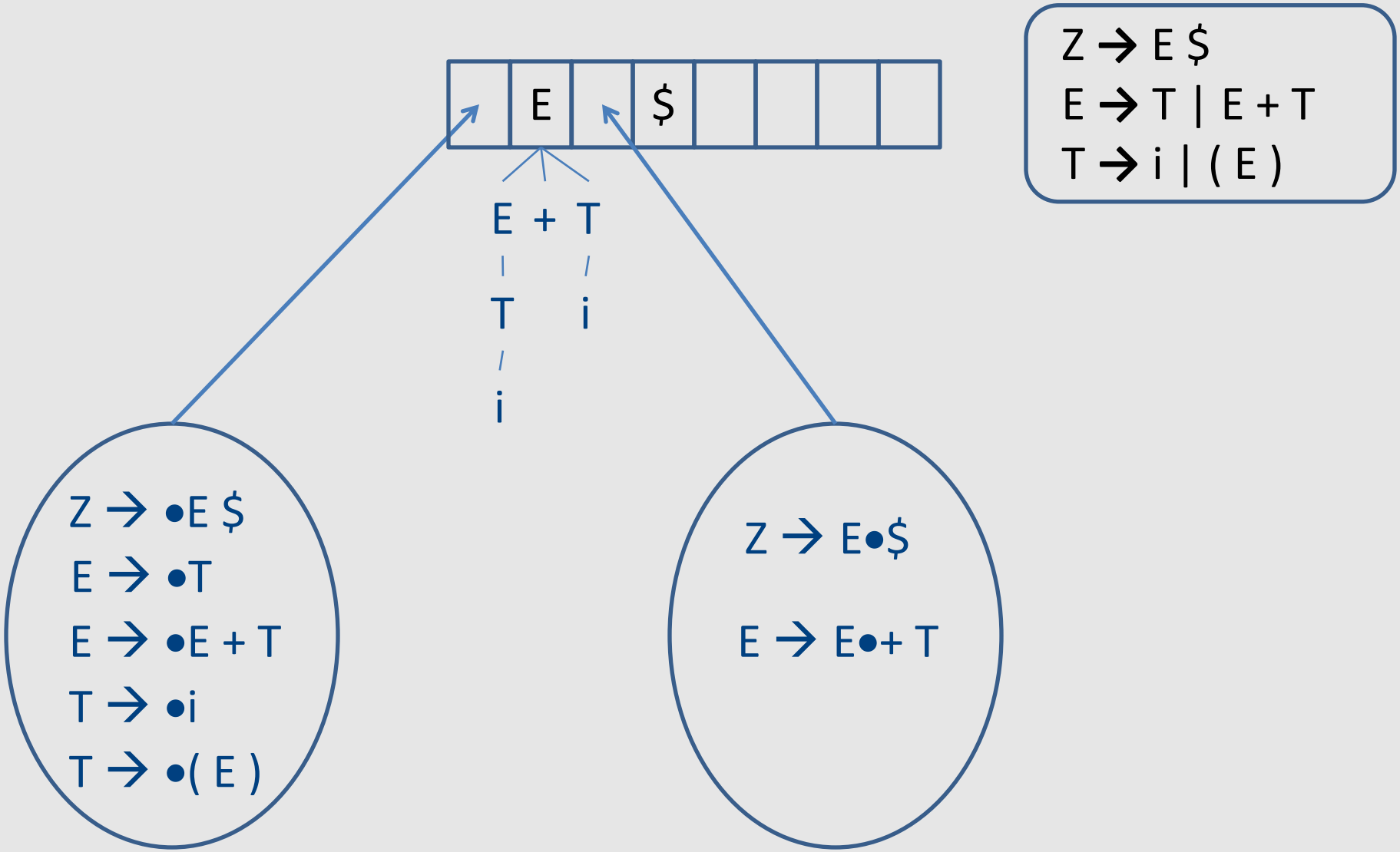
Example: parsing with LR items



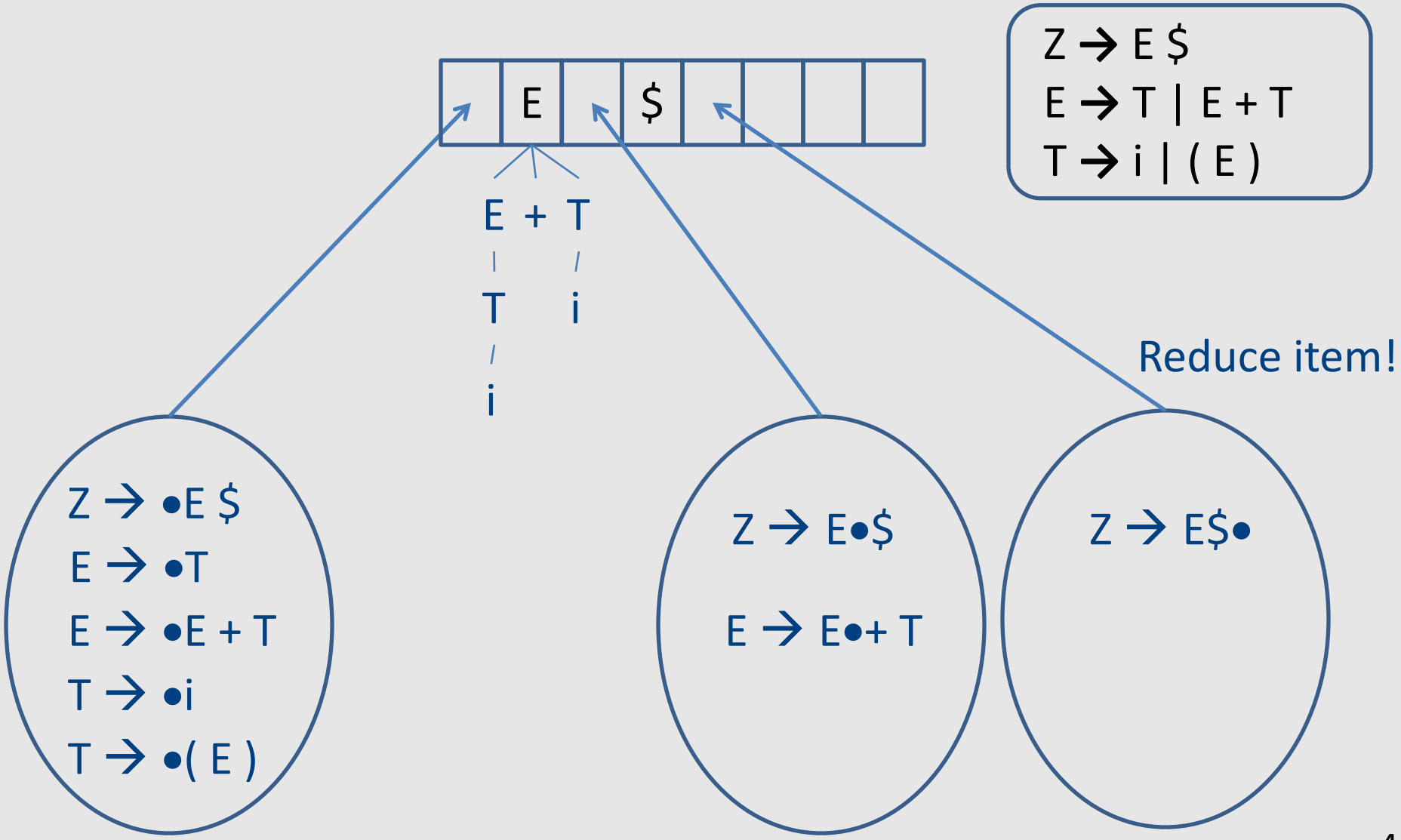
Example: parsing with LR items



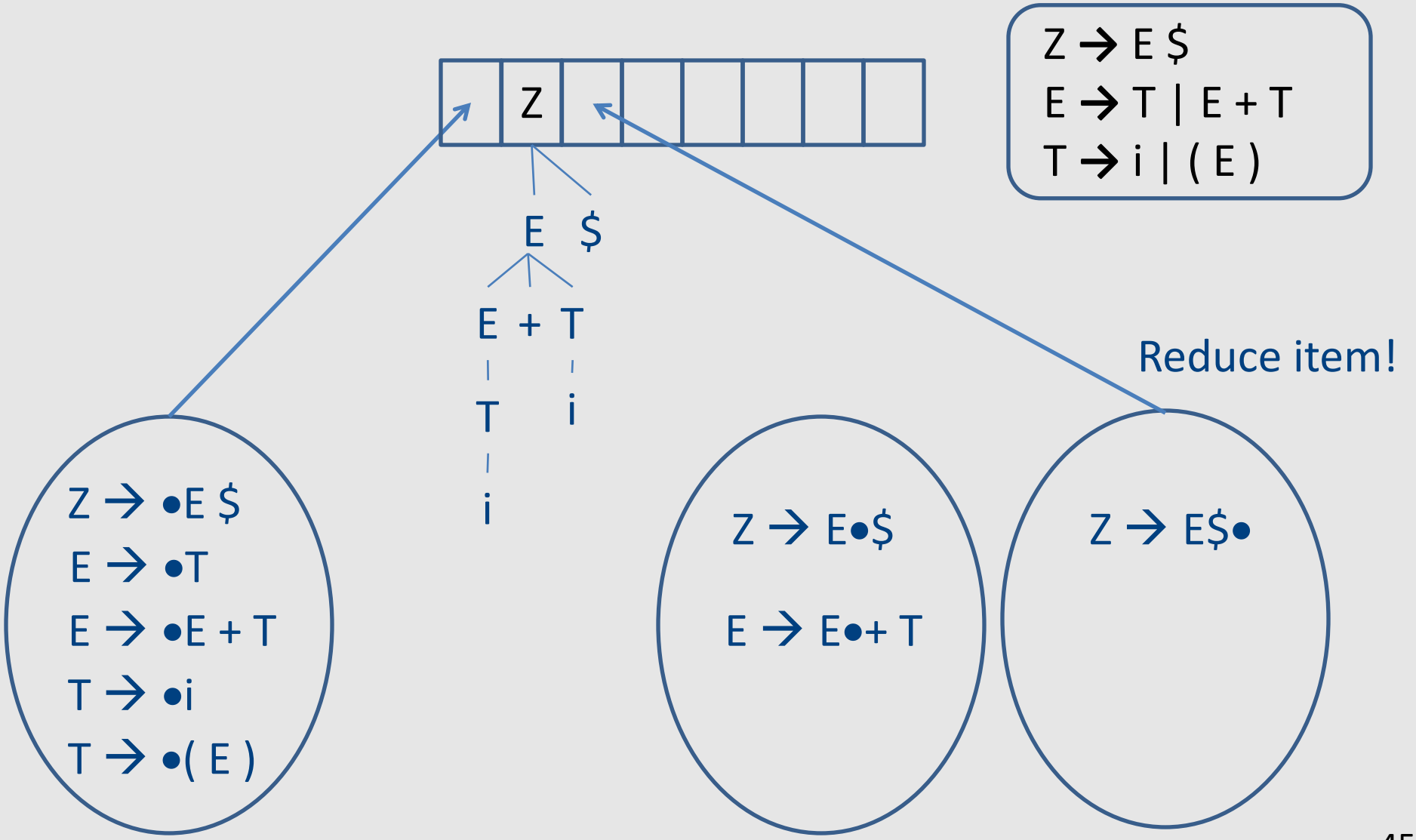
Example: parsing with LR items



Example: parsing with LR items



Example: parsing with LR items



GOTO/ACTION tables

empty – error move

GOTO Table

ACTION Table

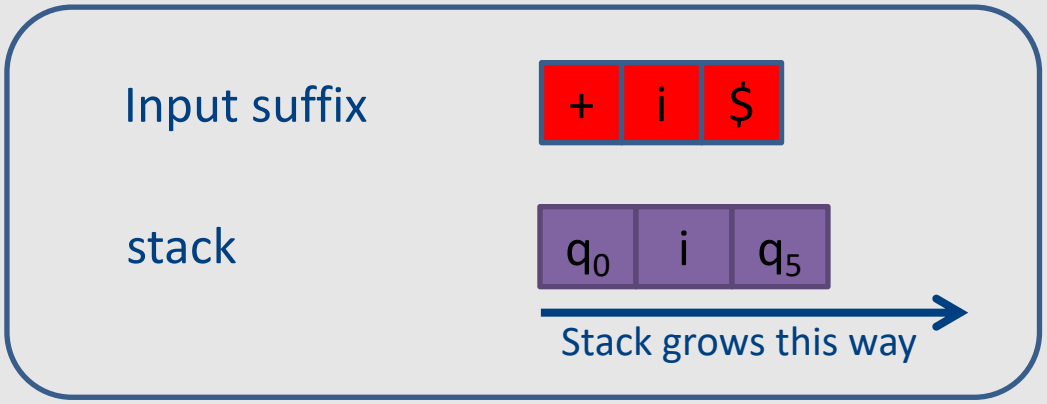
State	i	+	(	)	$	E	T	action
q0	q5		q7			q1	q6	shift
q1		q3			q2			shift
q2								Z→E$
q3	q5		q7				q4	Shift
q4								E→E+T
q5								T→i
q6								E→T
q7	q5		q7			q8	q6	shift
q8		q3		q9				shift
q9								T→E

LR(0) parser tables

- Two types of rows:
  - Shift row – tells which state to GOTO for current token
  - Reduce row – tells which rule to reduce (independent of current token)
    - GOTO entries are blank

LR parser data structures

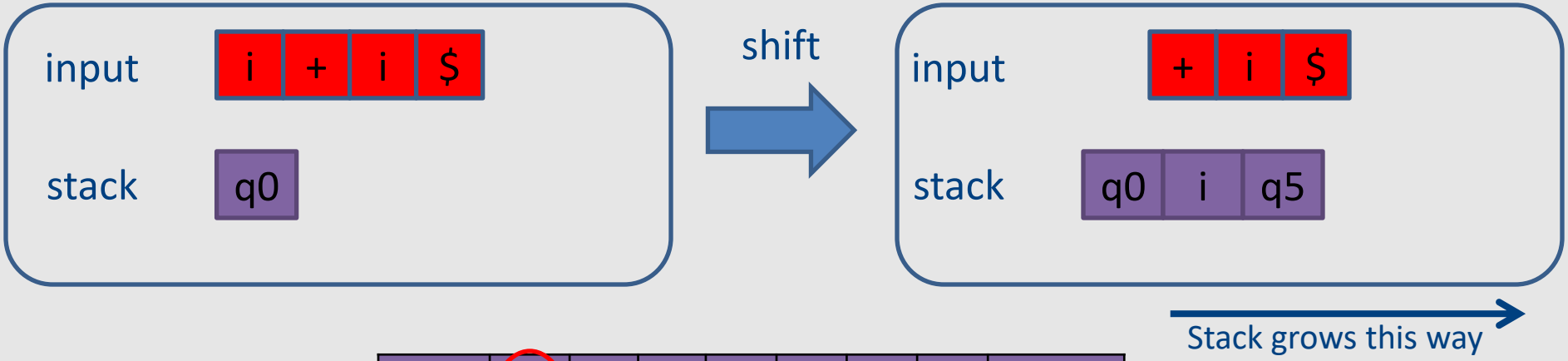
- Input – remainder of text to be processed
- Stack – sequence of pairs N, qi
  - N – symbol (terminal or non-terminal)
  - qi – state at which decisions are made



- Initial stack contains q0

LR(0) pushdown automaton

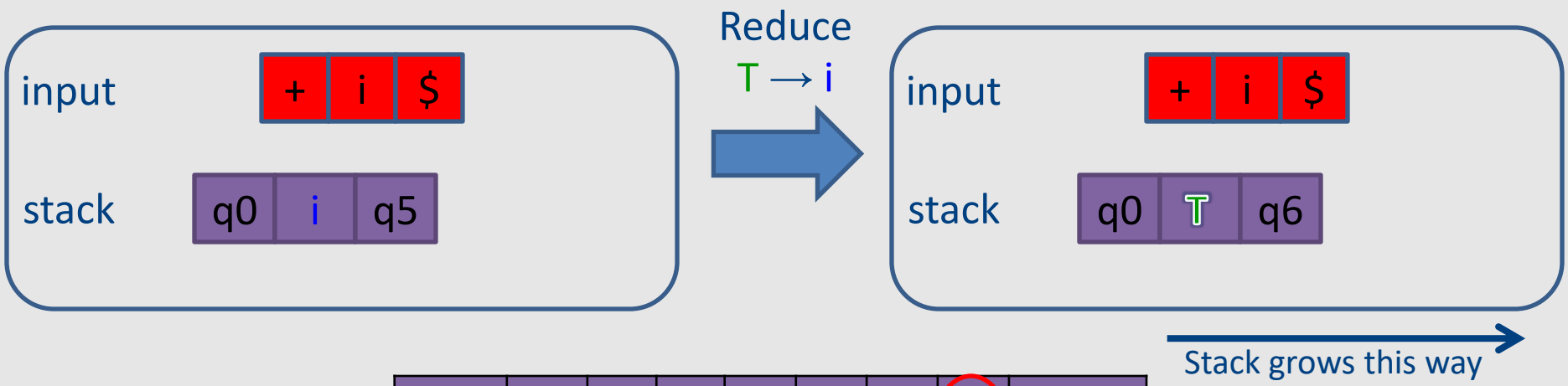
- Two moves: shift and reduce
- Shift move
  - Remove first token from input
  - Push it on the stack
  - Compute next state based on GOTO table
  - Push new state on the stack
  - If new state is error – report error



State	i	+	(	)	$	E	T	action
q0	q5		q7			q1	q6	shift

LR(0) pushdown automaton

- Reduce move
  - Using a rule N → α
  - Symbols in α and their following states are removed from stack
  - New state computed based on GOTO table (using top of stack, before pushing N)
  - N is pushed on the stack
  - New state pushed on top of N



State	i	+	(	)	$	E	T	action
q0	q5		q7			q1	q6	shift

GOTO/ACTION table

State	i	+	(	)	$	E	T
q0	s5		s7			s1	s6
q1		s3			s2		
q2	r1	r1	r1	r1	r1	r1	r1
q3	s5		s7				s4
q4	r3	r3	r3	r3	r3	r3	r3
q5	r4	r4	r4	r4	r4	r4	r4
q6	r2	r2	r2	r2	r2	r2	r2
q7	s5		s7			s8	s6
q8		s3		s9			
q9	r5	r5	r5	r5	r5	r5	r5

```
(1) Z → E $
(2) E → T
(3) E → E + T
(4) T → i
(5) T → ( E )
```

Warning: numbers mean different things!

rn = reduce using rule number n

sm = shift to state m

Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack grows this way

Stack	Input	Action
0	id + id $	s5

Initialize with state 0

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

rn = reduce using rule number n
sm = shift to state m

Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack grows this way

Stack	Input	Action
0	id + id $	s5

Initialize with state 0

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

rn = reduce using rule number n
sm = shift to state m

Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack grows this way

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

rn = reduce using rule number n
sm = shift to state m

Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack grows this way

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4

pop id 5

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

rn = reduce using rule number n
sm = shift to state m

Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack grows this way

Stack	Input	Action
0	id + id $	s5
0 ~~id 5~~	+ id $	r4

push T 6

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

rn = reduce using rule number n
sm = shift to state m

Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack grows this way

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

rn = reduce using rule number n
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Stack grows this way

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0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2
0 E 1	+ id $	s3

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

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Stack grows this way

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2
0 E 1	+ id $	s3
0 E 1 + 3	id $	s5

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

rn = reduce using rule number n
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Parsing id+id$

(1) S → E $
(2) E → T
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Stack grows this way

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2
0 E 1	+ id $	s3
0 E 1 + 3	id $	s5
0 E 1 + 3 id 5	$	r4

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

rn = reduce using rule number n
sm = shift to state m

Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
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Stack grows this way

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2
0 E 1	+ id $	s3
0 E 1 + 3	id $	s5
0 E 1 + 3 id 5	$	r4
0 E 1 + 3 T 4	$	r3

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

rn = reduce using rule number n
sm = shift to state m

Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
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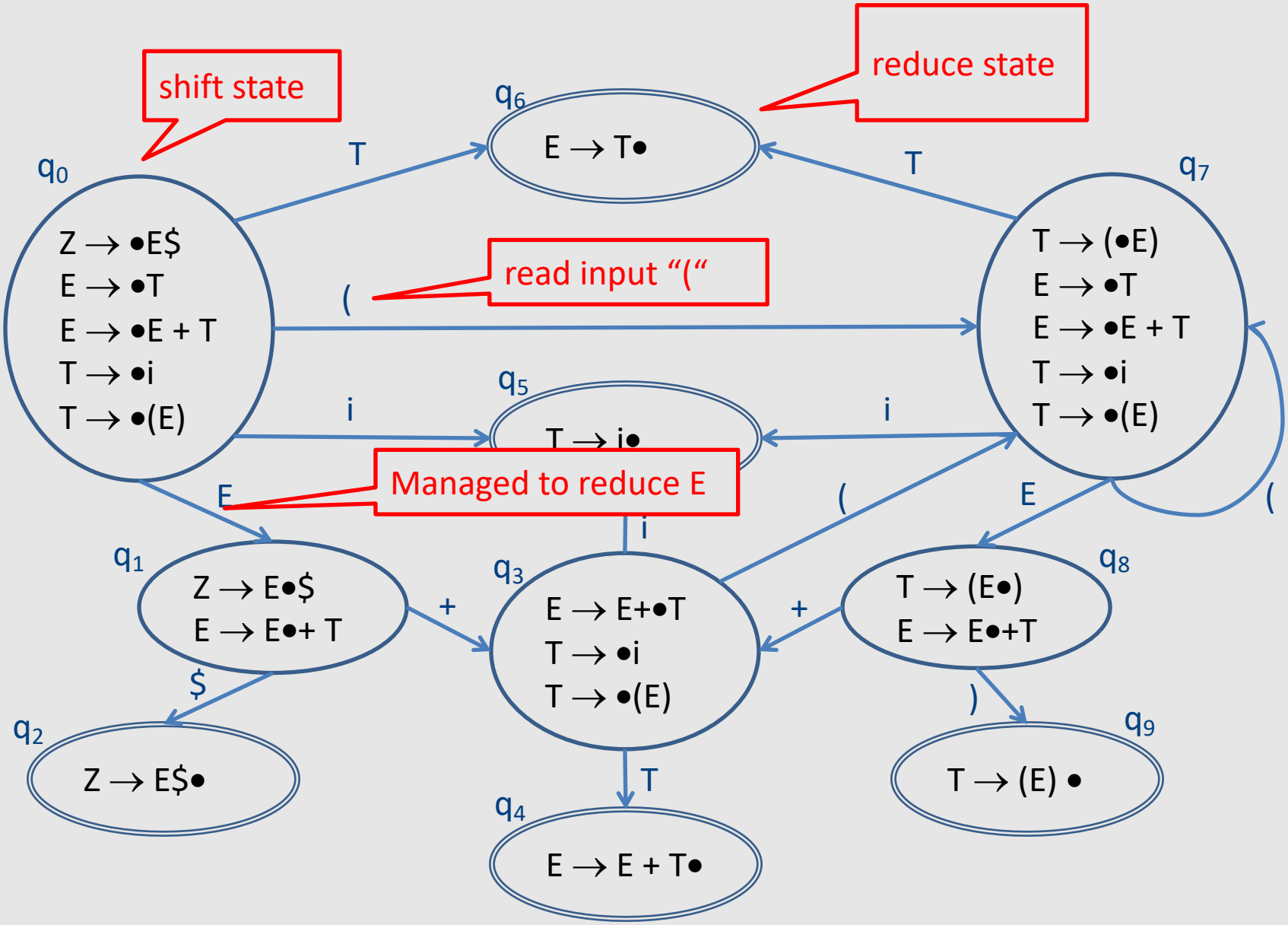
Stack grows this way

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2
0 E 1	+ id $	s3
0 E 1 + 3	id $	s5
0 E 1 + 3 id 5	$	r4
0 E 1 + 3 T 4	$	r3
0 E 1	$	s2

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

rn = reduce using rule number n
sm = shift to state m

LR(0) automaton example



States and LR(0) Items

$E \rightarrow E * B \mid E + B \mid B$
$B \rightarrow 0 \mid 1$

- The state will "remember" the potential derivation rules given the part that was already identified
- For example, if we have already identified E then the state will remember the two alternatives:

(1) $E \rightarrow E * B$, (2) $E \rightarrow E + B$

- Actually, we will also remember where we are in each of them:

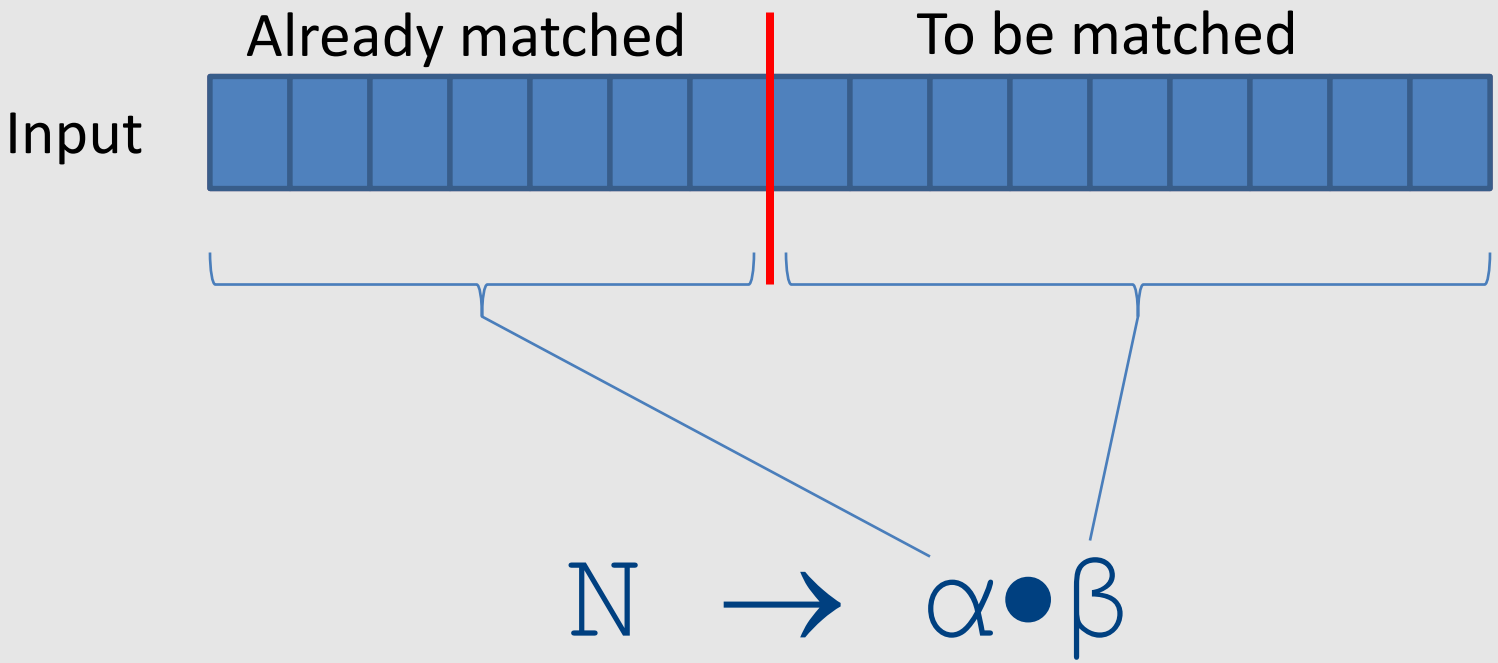
(1) $E \rightarrow E \bullet * B$, (2) $E \rightarrow E \bullet + B$

- A derivation rule with a location marker is called LR(0) item.
- The state is actually a set of LR(0) items. E.g., $q_{13} = \{ E \rightarrow E \bullet * B ,\ E \rightarrow E \bullet + B \}$

Constructing an LR parsing table

- Construct a (determinized) transition diagram from LR items
- If there are conflicts – stop
- Fill table entries from diagram

LR item



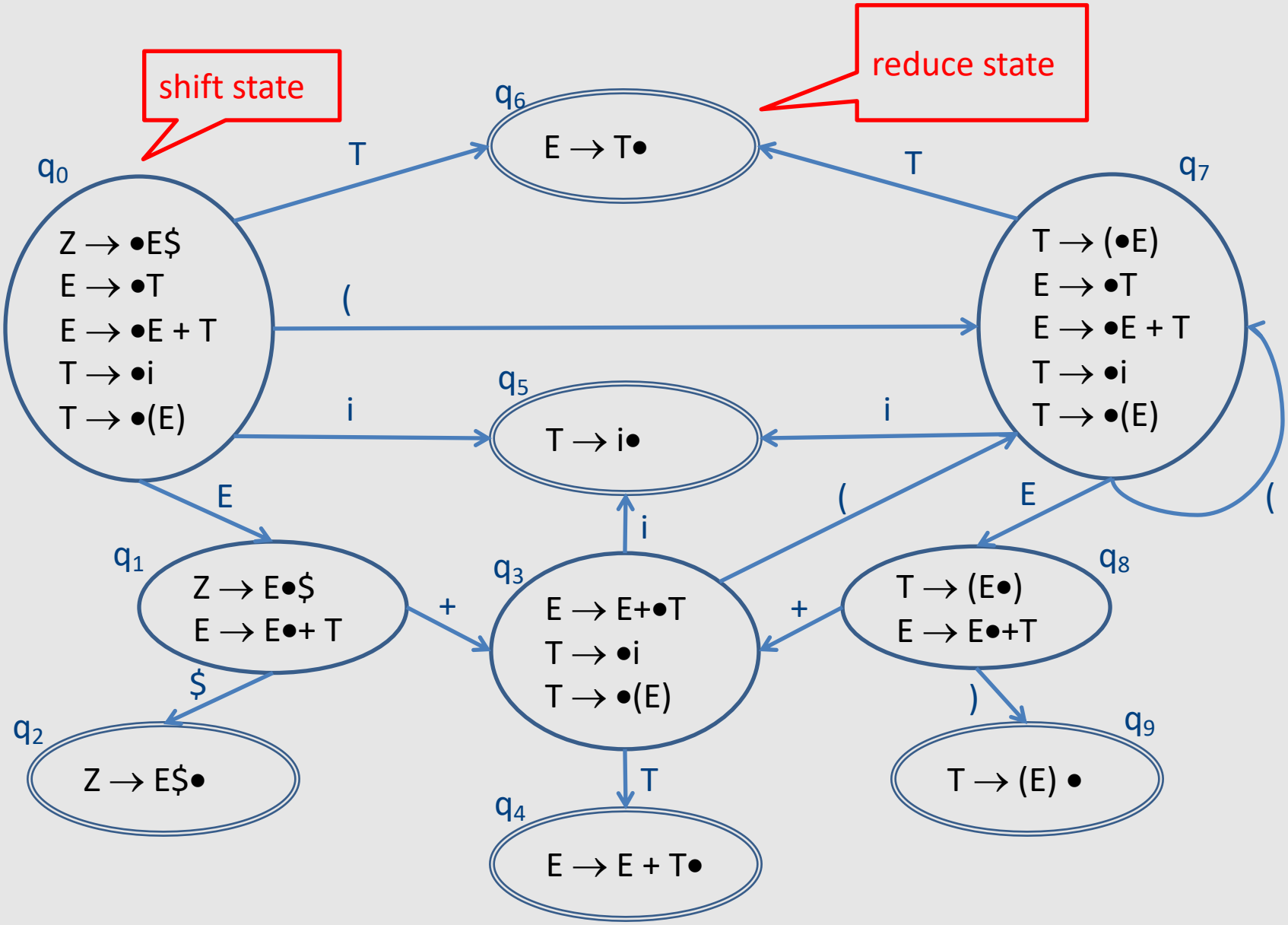
Hypothesis about αβ being a possible handle, so far we've matched α, expecting to see β

Types of LR(0) items

$N \rightarrow \alpha \bullet \beta$ Shift Item

$N \rightarrow \alpha \beta \bullet$ Reduce Item

LR(0) automaton example



Computing item sets

- Initial set
  - Z is in the start symbol
  - ε-closure({ $Z \rightarrow \bullet\alpha$ | $Z \rightarrow \alpha$ is in the grammar })

- Next set from a set S and the next symbol X
  - step(S,X) = { $N \rightarrow \alpha X \bullet \beta$ | $N \rightarrow \alpha \bullet X\beta$ in the item set S}
  - nextSet(S,X) = ε-closure(step(S,X))

Operations for transition diagram construction

- Initial = $\{S' \rightarrow \bullet S\$\}$

- For an item set I
  Closure(I) = Closure(I) $\cup$
  $\{X \rightarrow \bullet\mu$ is in grammar$|\ N \rightarrow \alpha\bullet X\beta$ in I$\}$

- Goto(I, X) = $\{\ N \rightarrow \alpha X\bullet\beta \mid N \rightarrow \alpha\bullet X\beta$ in I$\}$

Initial example

- Initial = {S $\rightarrow$ •E $}

Grammar

(1) S $\rightarrow$ E $
(2) E $\rightarrow$ T
(3) E $\rightarrow$ E + T
(4) T $\rightarrow$ id
(5) T $\rightarrow$ (E)

Closure example

- Initial = {S → •E $}
- Closure({S → •E $}) = {

  S → •E $

  E → •T

  E → •E + T

  T → •id

  T → •(E) }

Grammar

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Goto example

Grammar

(1) $S \rightarrow E\ \$$
(2) $E \rightarrow T$
(3) $E \rightarrow E + T$
(4) $T \rightarrow id$
(5) $T \rightarrow (\ E\ )$

- Initial = $\{S \rightarrow \bullet E\ \$\}$
- Closure($\{S \rightarrow \bullet E\ \$\}$) = {

  $S \rightarrow \bullet E\ \$$

  $E \rightarrow \bullet T$

  $E \rightarrow \bullet E + T$

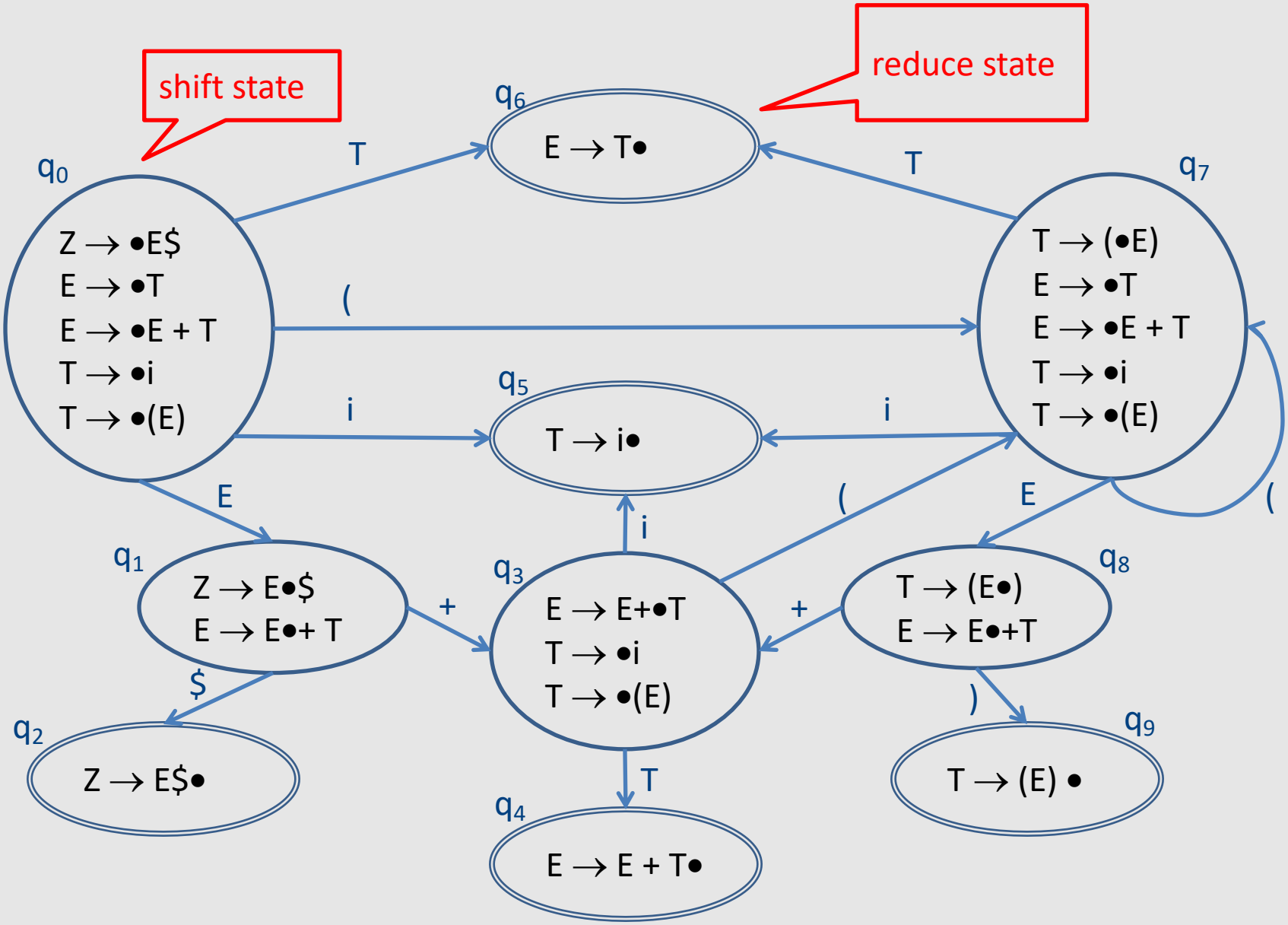
  $T \rightarrow \bullet id$

  $T \rightarrow \bullet (\ E\ )$ }
- Goto($\{S \rightarrow \bullet E\ \$\ ,\ E \rightarrow \bullet E + T,\ T \rightarrow \bullet id\}$, E) = $\{S \rightarrow E \bullet\ \$,\ E \rightarrow E \bullet + T\}$

Constructing the transition diagram

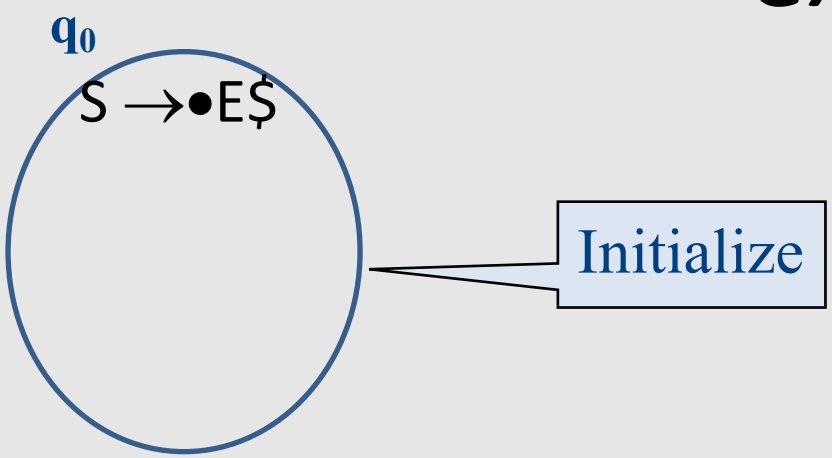
- Start with state 0 containing item Closure({S $\rightarrow$ •E $})
- Repeat until no new states are discovered
  - For every state p containing item set Ip, and symbol N, compute state q containing item set Iq = Closure(goto(Ip, N))

LR(0) automaton example



Automaton construction example

(1) $S \rightarrow E\ \$$
(2) $E \rightarrow T$
(3) $E \rightarrow E + T$
(4) $T \rightarrow id$
(5) $T \rightarrow (\ E\ )$



Automaton construction example

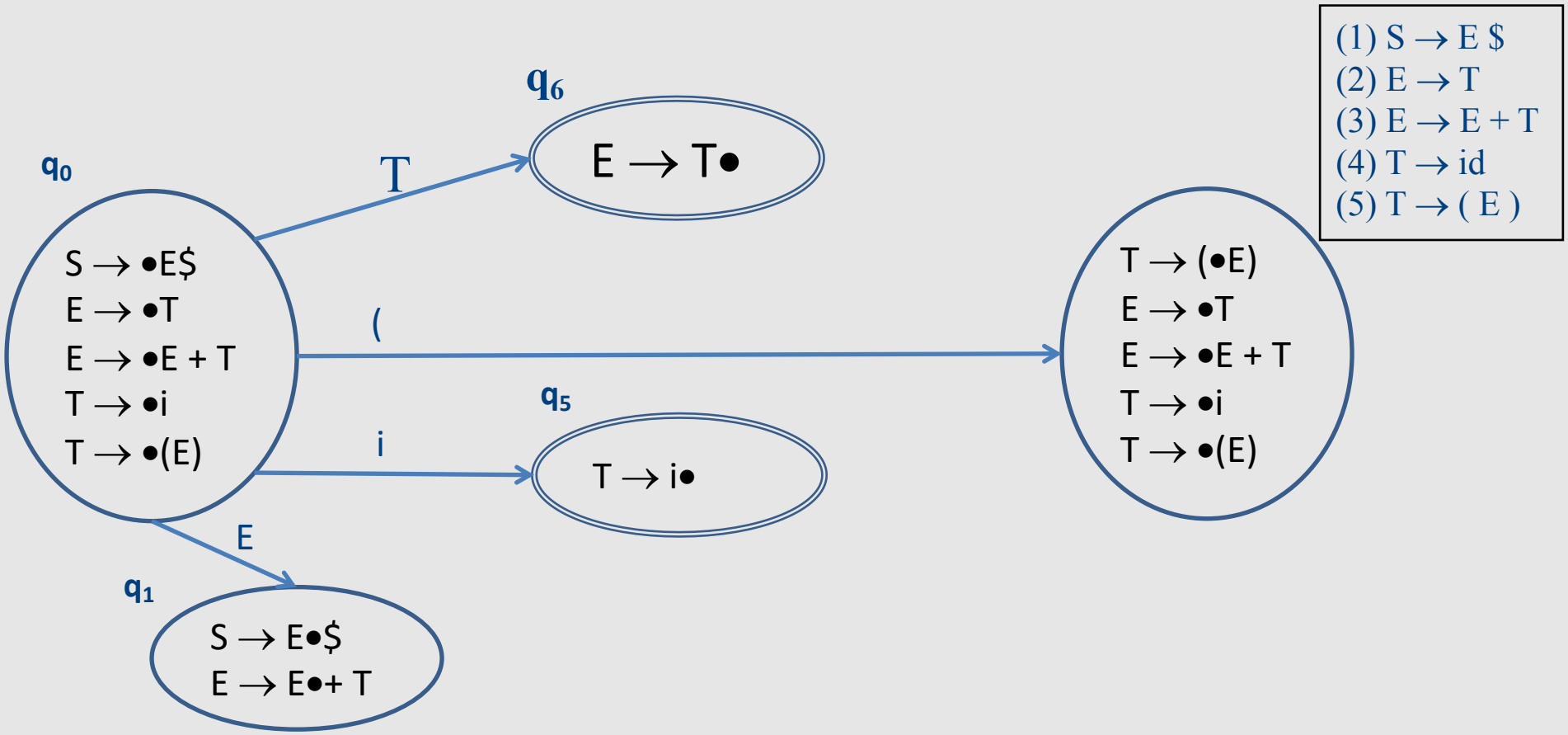
(1) $S \rightarrow E\ \$$
(2) $E \rightarrow T$
(3) $E \rightarrow E + T$
(4) $T \rightarrow id$
(5) $T \rightarrow (\ E\ )$

q_0

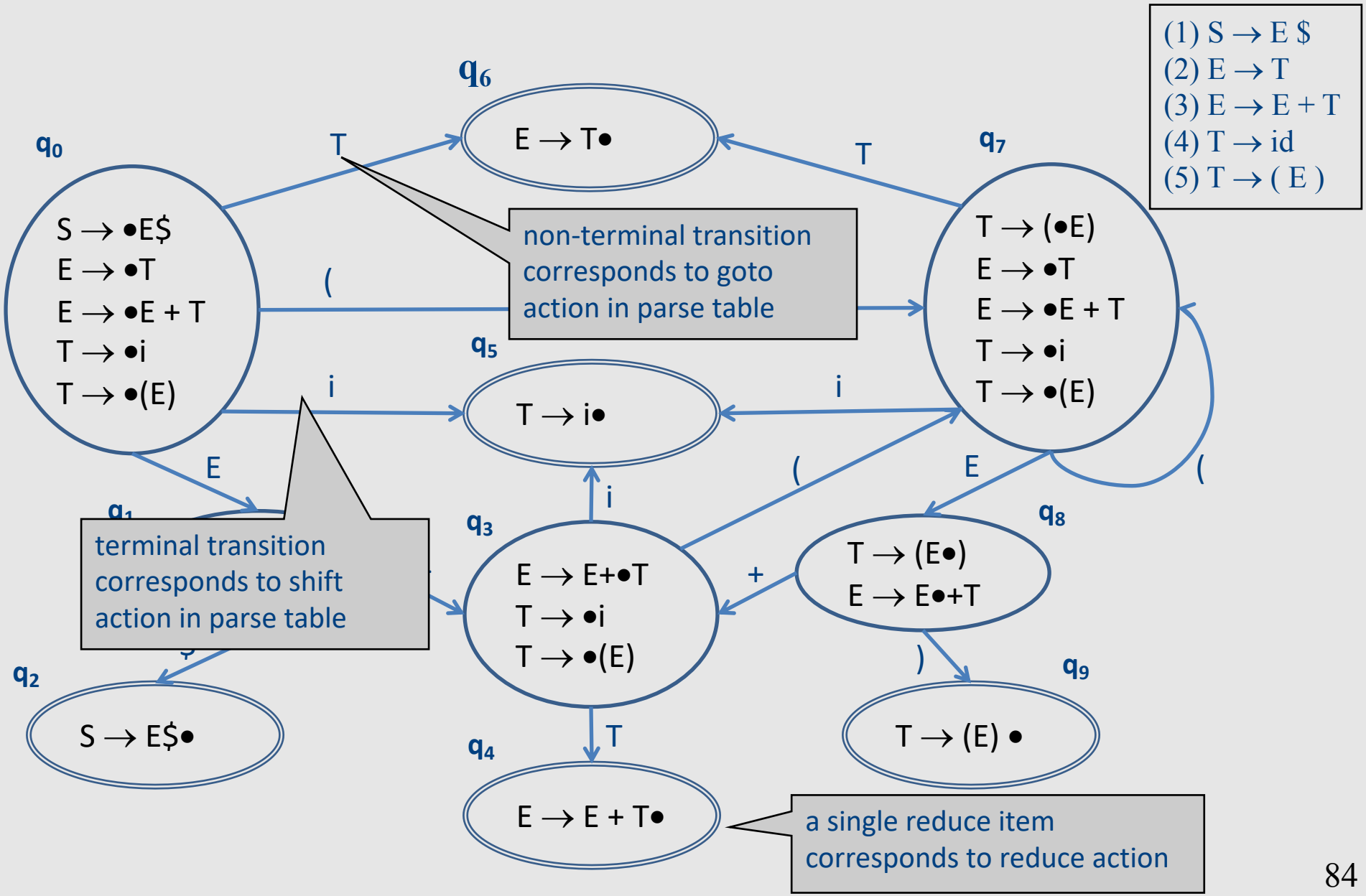
$S \rightarrow \bullet E\$$
$E \rightarrow \bullet T$
$E \rightarrow \bullet E + T$
$T \rightarrow \bullet i$
$T \rightarrow \bullet (E)$

apply Closure

Automaton construction example



Automaton construction example

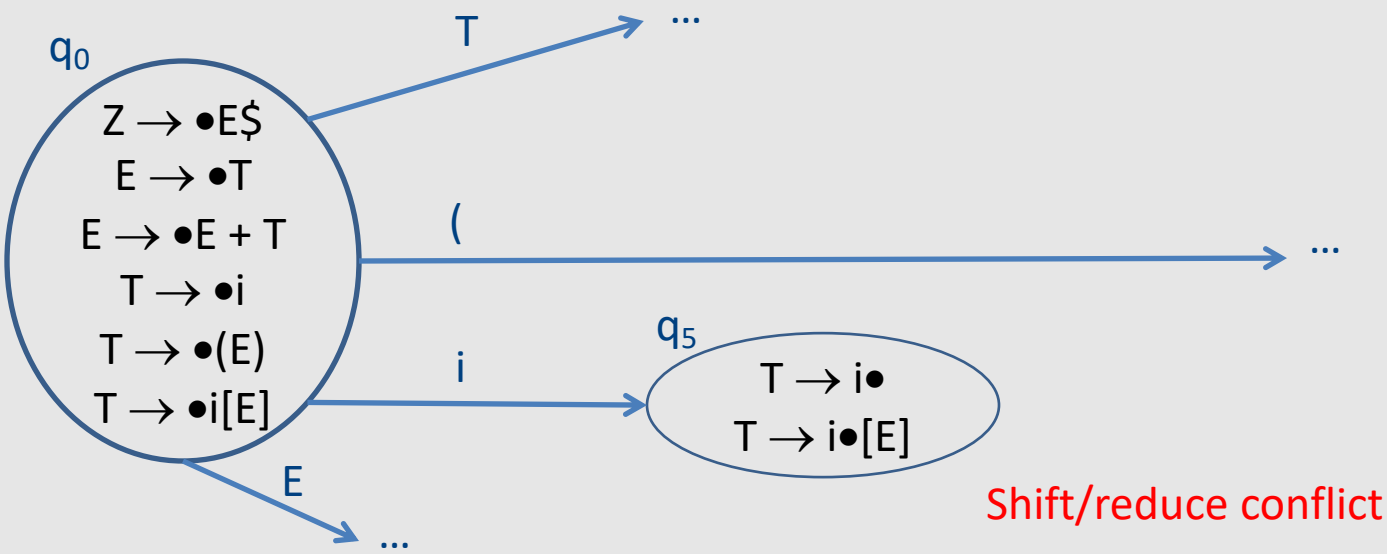


Are we done?

- Can make a transition diagram for any grammar
- Can make a GOTO table for every grammar

- Cannot make a deterministic ACTION table for every grammar

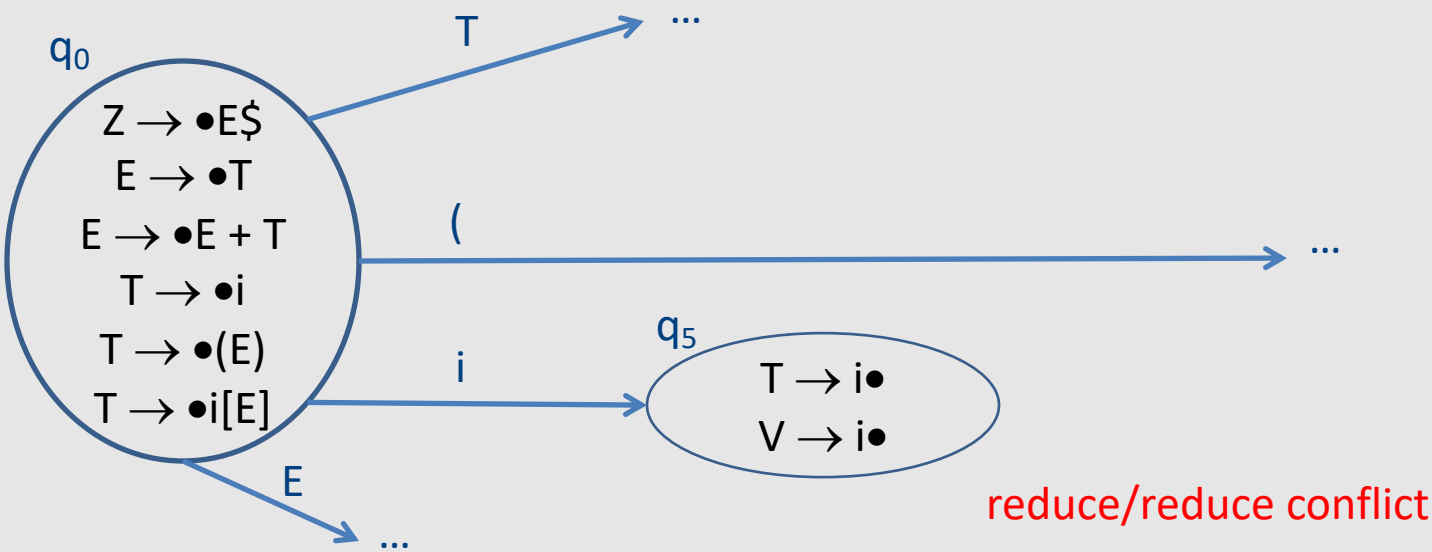
LR(0) conflicts



Shift/reduce conflict

Z → E $
E → T
E → E + T
T → i
T → (E)
T → i[E]

LR(0) conflicts



reduce/reduce conflict

Z → E $
E → T
E → E + T
T → i
V → i
T → (E)

LR(0) conflicts

- Any grammar with an ε-rule cannot be LR(0)
- Inherent shift/reduce conflict
  - $A \rightarrow \varepsilon\bullet$ – reduce item
  - $P \rightarrow \alpha\bullet A\beta$ – shift item
  - $A \rightarrow \varepsilon\bullet$ can always be predicted from $P \rightarrow \alpha\bullet A\beta$

Conflicts

- Can construct a diagram for every grammar but some may introduce conflicts
- shift-reduce conflict: an item set contains at least one shift item and one reduce item
- reduce-reduce conflict: an item set contains two reduce items

LR variants

- LR(0) – what we've seen so far
- SLR(0)
  - Removes infeasible reduce actions via FOLLOW set reasoning
- LR(1)
  - LR(0) with one lookahead token in items
- LALR(0)
  - LR(1) with merging of states with same LR(0) component

LR (0) GOTO/ACTIONS tables

GOTO table is indexed by state and a grammar symbol from the stack

GOTO Table

ACTION Table

State	i	+	(	)	$	E	T	action
q0	q5		q7			q1	q6	shift
q1		q3			q2			shift
q2								Z→ E$
q3	q5		q7				q4	Shift
q4								E→ E+T
q5								T→ i
q6								E→ T
q7	q5		q7			q8	q6	shift
q8		q3		q9				shift
q9								T→ E

ACTION table determined <u>only</u> by state, <u>ignores input</u>

SLR parsing

- A handle should not be reduced to a non-terminal N if the lookahead is a token that cannot follow N
- A reduce item $N \rightarrow \alpha\bullet$ is applicable only when the lookahead is in FOLLOW(N)
  - If b is not in FOLLOW(N) we proved there is no derivation $S \Rightarrow^* \beta N b$.
  - Thus, it is safe to remove the reduce item from the conflicted state
- Differs from LR(0) only on the ACTION table
  - Now a row in the parsing table may contain both shift actions and reduce actions and we need to consult the current token to decide which one to take

SLR action table

Lookahead token from the input

State	i	+	(	)	[	]	$
0	shift		shift				
1		shift					accept
2							
3	shift		shift				
4		E→ E+T		E→ E+T			E→ E+T
5		T→ i		T→ i	**shift**		T→ i
6		E→ T		E→ T			E→ T
7	shift		shift				
8		shift		shift			
9		T→ (E)		T→ (E)			T→ (E)

vs.

state	action
q0	shift
q1	shift
q2	
q3	shift
q4	E→ E+T
q5	T→ i
q6	E→ T
q7	shift
q8	shift
q9	T→ E

SLR – use 1 token look-ahead

LR(0) – no look-ahead

```
… as before…
T → i
T → i[E]
```

LR(1) grammars

- In SLR: a reduce item $N \rightarrow \alpha\bullet$ is applicable only when the lookahead is in FOLLOW(N)
- But FOLLOW(N) merges lookahead for all alternatives for N
  - Insensitive to the context of a given production

- LR(1) keeps lookahead with each LR item
- Idea: a more refined notion of follows computed per item

LR(1) items

- LR(1) item is a pair
  - LR(0) item
  - Lookahead token
- Meaning
  - We matched the part left of the dot, looking to match the part on the right of the dot, followed by the lookahead token
- Example
  - The production L → id yields the following LR(1) items

(0) S' → S$
(1) S → L = R
(2) S → R
(3) L → * R
(4) L → id
(5) R → L

LR(0) items

[L → ● id]
[L → id ●]

LR(1) items

[L → ● id, *]
[L → ● id, =]
[L → ● id, id]
[L → ● id, $]
[L → id ●, *]
[L → id ●, =]
[L → id ●, id]
[L → id ●, $]

LR(1) items

- LR(1) item is a pair
  - LR(0) item
  - Lookahead token
- Meaning
  - We matched the part left of the dot, looking to match the part on the right of the dot, followed by the lookahead token
- Example
  - The production L $\rightarrow$ id yields the following LR(1) items

- Reduce only if the the expected lookhead matches the input
  - [L → id ●, =] will be used only if the next input token is =

LALR(1)

- LR(1) tables have huge number of entries
- Often don’t need such refined observation (and cost)
- Idea: find states with the same LR(0) component and merge their lookaheads component as long as there are no conflicts
- LALR(1) not as powerful as LR(1) in theory but works quite well in practice
  - Merging may not introduce new shift-reduce conflicts, only reduce-reduce, which is unlikely in practice

Summary

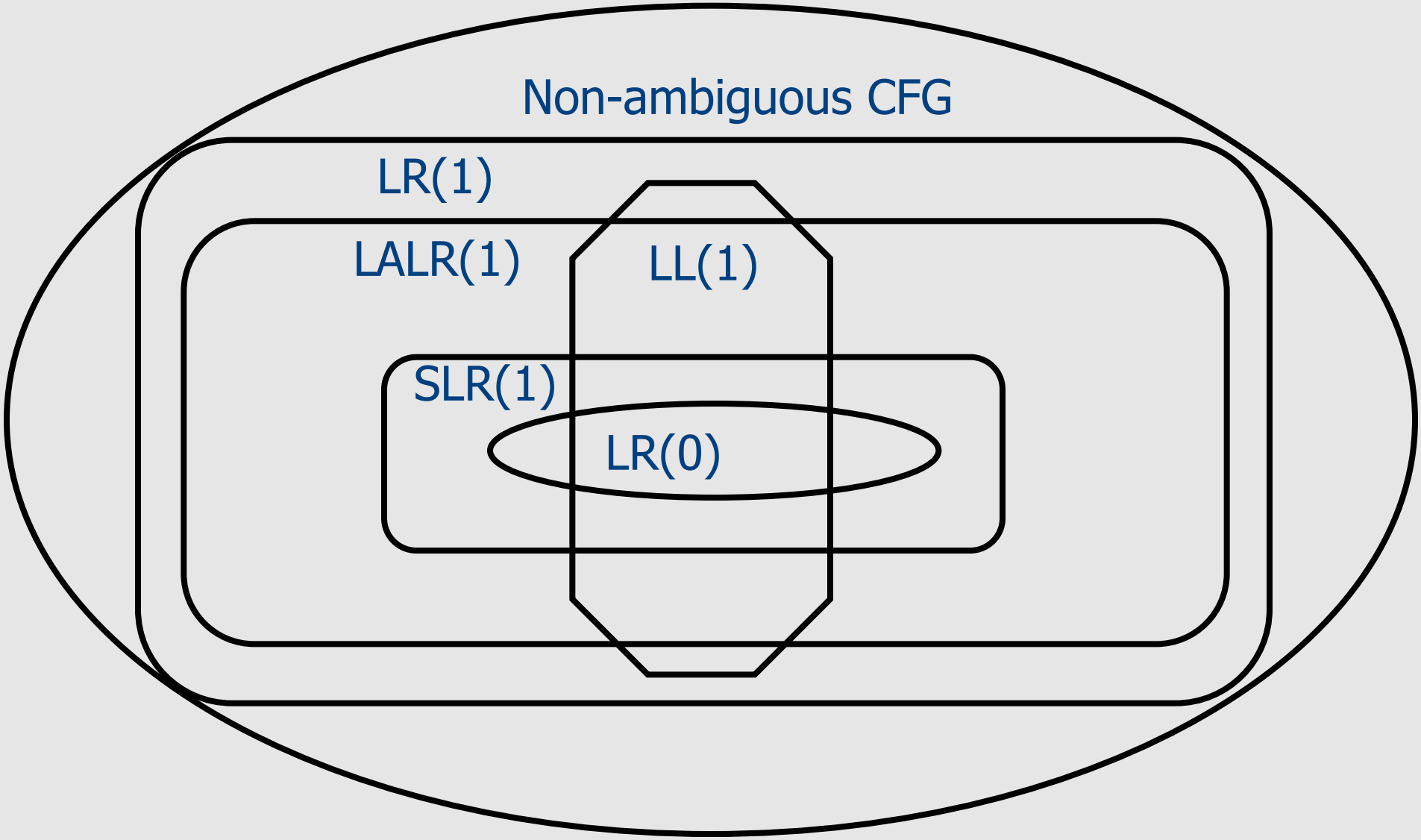
LR is More Powerful than LL

- Any LL(k) language is also in LR(k), i.e., LL(k) ⊂ LR(k).
  - LR is more popular in automatic tools
- But less intuitive
- Also, the lookahead is counted differently in the two cases
  - In an LL(k) derivation the algorithm sees the left-hand side of the rule + k input tokens and then must select the derivation rule
  - In LR(k), the algorithm "sees" all right-hand side of the derivation rule + k input tokens and then reduces
    - LR(0) sees the entire right-side, but no input token

Using tools to parse + create AST

```
terminal Integer NUMBER;
terminal PLUS,MINUS,MULT,DIV;
terminal LPAREN, RPAREN;
terminal UMINUS;
nonterminal Integer expr;
precedence left PLUS, MINUS;
precedence left DIV, MULT;
Precedence left UMINUS;
%%
expr ::= expr:e1 PLUS expr:e2
        {: RESULT = new Integer(e1.intValue() + e2.intValue()); :}
        | expr:e1 MINUS expr:e2
        {: RESULT = new Integer(e1.intValue() - e2.intValue()); :}
        | expr:e1 MULT expr:e2
        {: RESULT = new Integer(e1.intValue() * e2.intValue()); :}
        | expr:e1 DIV expr:e2
        {: RESULT = new Integer(e1.intValue() / e2.intValue()); :}
        | MINUS expr:e1 %prec UMINUS
        {: RESULT = new Integer(0 - e1.intValue(); :}
        | LPAREN expr:e1 RPAREN
        {: RESULT = e1; :}
        | NUMBER:n
        {: RESULT = n; :}
```

Grammar Hierarchy




That's all Folks