Graph and hypergraph coloring

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1. Basic definitions: vertex coloring, chromatic number, edge coloring, chromatic index. Coloring infinite graphs, De Brujin-Erdős theorem
2. Vertex degrees and colorings. Brooks' theorem, degeneracy and chromatic number
3. Color-critical graphs. Structure of color-critical graphs, Gallai's theorem. Extremal problems on color-critical graphs, sparse and dense color-critical graphs. Constructions of color-critical graphs. Universality of Hajós' construction.
4. Coloring graphs on surfaces. Heawood theorem. Four-color theorem. Hadwiger conjecture
5. Perfect graphs. Classes of perfect graphs. Lovasz' theorem
6. Coloring random graphs. Asymptotic value of the chromatic number of random graphs. Performance of the greedy algorithm on random graphs. Hajós' and Hadwiger's conjecture for random graphs.
7. Sparse graphs. Graphs with high girth and high chromatic number
8. Hypergraph coloring. Property B and theorems of Erdős and of Radhakrishnan and Srinivasan. Applications of the Local Lemma
9. List coloring. Degrees and choice number. Choosability in bipartite graphs. Five-choosability of planar graphs. Algebraic techniques. Choice number of random graphs
10. Chromatic polynomial. Linear algebraic techniques. Eigenvalues and chromatic number
11. Edge coloring. Theorems of König, Vizing and Shannon. List coloring conjecture. Dinitz' conjecture and its resolution by Galvin
12. Algorithmic issues. NP-completeness of graph and hypergraph coloring problems. Approximate graph coloring. Coloring 3-colorable graphs. Coloring 2-colorable hypergraphs