



# Cell Identification Codes for Tracking Mobile Users

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**Abstract.** The minimization of the wireless cost of tracking mobile users is a crucial issue in wireless networks. Some of the previous strategies addressing this issue leave an open gap, by requiring the use of information that is not generally available to the user (for example, the distance traveled by the user). For this reason, both the implementation of some of these strategies and the performance comparison to existing strategies is not clear. In this work we propose to close this gap by the use of *Cell Identification Codes* (CIC) for tracking mobile users. Each cell periodically broadcasts a short message which identifies the cell and its orientation relatively to other cells in the network. This information is used by the users to efficiently update their location. We propose several cell identification encoding schemes, which are used to implement different tracking strategies, and analyze the amount of information required by each tracking strategy. One of our major results is that there is no need to transmit a code which is unique for each cell. For example, a 3 bits CIC is sufficient to implement a distance-based tracking strategy in a two-dimensional system. In addition, we propose a combination of timer and movement tracking strategy, based on either a one-bit or a two-bit CIC, depending on system topology and user mobility. An important property of our framework is that the *overall* performance cost, and hence its comparison to existing methods, is evaluated for each tracking strategy. The CIC-based strategies are shown to outperform the geographic-based method currently used in existing networks, and the timer-based method, over a wide range of parameters. Moreover, this superiority increases as the number of users per cell increases.

**Keywords:** PCS, wireless, mobile, user tracking

## 1. Introduction

The increasing demand for personal communication services (PCS) will require future wireless networks to gracefully accommodate mobility of both users and services. Contrary to the wired networks, in which user location is fixed, in wireless networks a user can potentially be located anywhere within the system service area. As the number of mobile users keeps increasing, the amount of signaling traffic required for location management keeps growing. The cost associated with the need to locate a mobile user is composed of two parts: (1) the cost of accessing data bases, such as Home Location Register (HLR) and Visitor Location Register (VLR), and (2) the cost of radio signaling over the control channel. The issue considered in this study is the utilization of the *wireless* resources associated with tracking.

Existing cellular systems use the following tracking strategy, known as the geographic-based strategy. The geographic area is partitioned into *location areas*, based on the commercial licenses granted to the operating companies. A location area (LA) is a group of cells, referred to as a home-system. The term *location area* is used by GSM systems, while IS-41 refers to the LA as *registration area*. Users register whenever they change LA, while within the LA they never register. The implementation of the geographic-based (GB) strategy is very simple: all base stations within the same LA periodically broadcast the ID of the LA. Each user compares its last LA ID with the current ID, and transmits a registration message whenever the ID changes. Hence, the user is not aware of its exact location within the LA. When there is an incoming call directed to a user, all the cells within its current LA are paged. Since the number of

cells within a typical LA is very large, the tracking cost associated with the GB strategy is very high. Hence, there is a need to provide more accurate location information to the users.

In this paper we propose a new approach for providing the users with location information, or other related information, necessary to reduce the wireless cost of tracking. The basic idea is to use (in addition to the LA ID) a *Cell Identification Code* (CIC), which identifies the cells and their relative orientation. Each cell periodically broadcasts its identification code through the down link control channel (for example, DCCCH in GSM systems). The goal of the CIC is to provide the location information required to the users, in order to perform the registration strategy.

The first issue addressed in this paper is the proposal of efficient cell identification encoding schemes, each geared towards a different tracking strategy. We propose three CIC encoding schemes. The first, achieving the best performance, is the proposition of a CIC for implementing a distance-based tracking strategy. We show that for a realistic two dimensional topology a four bit message is sufficient to provide a reliable distance-based tracking. We analyze the conditions under which the proposed CIC is guaranteed to properly function. Secondly, we propose a combined timer and movement tracking strategy, based on either a one-bit or a two-bit CIC, depending on system topology and user mobility. We show that the movement-based tracking strategy is a special case of the combined timer and movement tracking strategy. The third proposition is a conditional timer strategy, in which the user examines its current location every  $T$  time units, using a CIC, and if it differs from its last known location – it transmits a registration message.

The second issue addressed in this paper is the evaluation of the reduction in paging cost, in comparison to the additional cost incurred by the CIC transmission. We show that the *overall* performance of the CIC-based strategies is superior to that of the timer-based method, being used in existing cellular networks, over a wide range of parameters.

The main results of this study are:

1. Introducing the concept of Cell Identification Codes (CIC).
2. The analysis of the amount of information required for the implementation of various tracking strategies.
3. The suggestion of new tracking strategies, such as the conditional timer and the combined timer and movement tracking strategies.
4. The implementations of tracking strategies suggested before, such as the distance-based strategy, whose implementation had never been considered before.
5. The performance comparison to existing strategies, which takes into consideration the implementation cost of each strategy.

The structure of this paper is as follows. In section 2 we suggest efficient cell identification encoding schemes for various tracking strategies, and establish their correctness. In section 3 we analyze the performance of the tracking strategies for a two-dimensional grid system. A numerical comparison between the various methods (achieved by a performance evaluation) is provided in section 4. Summary and concluding remarks are given in section 5.

## 2. The Cell Identification Codes (CIC)

Below we describe several cell identification encoding schemes which support different tracking strategies, and the associated wireless cost of their implementation. Each cell is assigned a code, which identifies its orientation relatively to other cells within the same LA. The code does not have to be unique for each cell. In fact, the best performance is achieved by a four-bit CIC, used to implement a distance-based tracking strategy in a two-dimensional system.

We consider a wireless network partitioned into cells. The user location is understood as the cell in which the user is currently residing. Two cells are called neighboring if a user can move from one to another without crossing any other cell.

### 2.1. The distance-based strategy

The idea of distance-based tracking strategy was first suggested in [3] and [6], and it is considered as the most efficient tracking strategy [3]. The basic idea is that whenever the distance, measured in cells, between the user last known location and its current location exceeds a threshold distance, say  $D$ , the user registers. Hence, in order to implement the distance-based strategy, the user must be able to calculate the distance between its current location and its last known location. To the best of our knowledge, the implementation issue

(0,0)	(0,1)	(0,2)	(0,0)	(0,1)
(1,0)	(1,1)	(1,2)	(1,0)	(1,1)
(2,0)	(2,1)	(2,2)	(2,0)	(2,1)
(0,0)	(0,1)	(0,2)	(0,0)	(0,1)
(1,0)	(1,1)	(1,2)	(1,0)	(1,1)

Figure 1. CIC-D encoding for distance based strategy, using rectangle tiles.

of this strategy has not been addressed. Unfortunately, the required location information is not generally available to the user. For this reason, this strategy is not supported by existing cellular networks. The distance between any pair of cells, in terms of number of cells, depends in general on the system topology. We therefore assume that the system topology is the following. The network is partitioned into location areas. Each location area (LA) is a group of two-dimensional cells. The system coverage area is assumed to be a continuous area, namely, containing no “holes”. That is, given any two cells, say  $x$  and  $y$ , the shortest path between  $x$  and  $y$  is completely contained within the system. To the best of our knowledge, these assumptions hold in practice for most of the wireless networks.

#### 2.1.1. The encoding scheme

The cell identification code proposed for the distance based strategy, CIC-distance, or CIC-D in short, is as follows. A uniform grid is superposed on the plane. The identity of any tile on the grid is the Cartesian product of the indices of the horizontal and vertical bands within which the tile lies. The bands are indexed using a modulo-3 increasing sequence, in a predefined direction (for example: East–West and North–South). The basic idea is that a modulo-3 indexing uniquely identifies the motion direction: the CIC index always increases along one direction, and decreases along the opposite direction. Consequently, the user can easily compute the distance traveled by tracking a modulo-3 CIC indexing. Figure 1 depicts the indexing of tiles on the grid. Along each dimension a cell is identified by the band on which that cell spans. If a cell spans on more than one band (for example, the cell spans on two neighboring bands), then that cell is identified by the first of these bands along the predefined direction mentioned above. For example, the vertical cell code is determined by the northernmost tile on which that cell spans. Figure 2 demonstrates this indexing for hexagonal shaped cells. When a paging event occurs, the search is conducted first at the user last reported cell. If it is not found there, then the search is conducted simultaneously at all cells associated with the same tile as the last reported cell. If the user is not found at the last reported tile, then the search is conducted in increasing distance order from this tile, until the user is found.

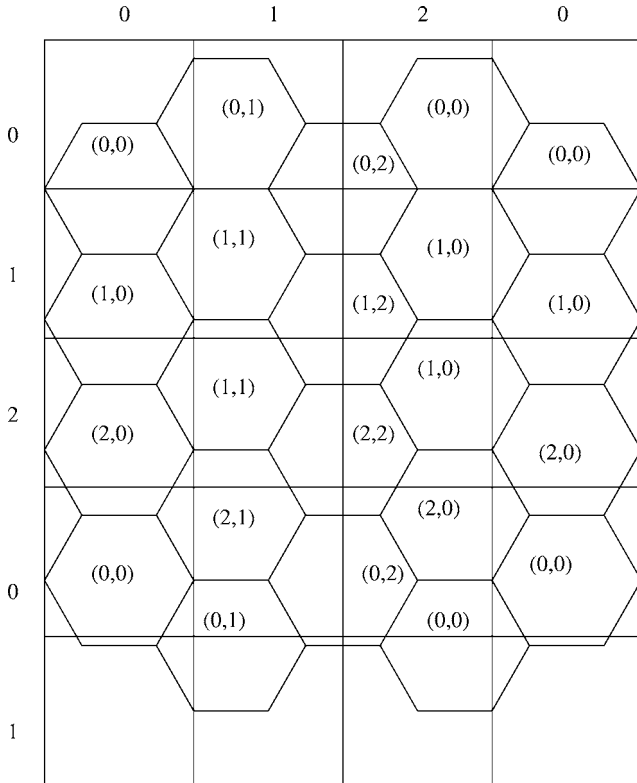


Figure 2. CIC-D encoding for hexagonal shaped cells, using rectangle tiles. The cell code is determined by the top and rightmost tile on which that cell spans.

The number of tiles being paged simultaneously depends on the paging delay constraint. For a two-dimensional system the CIC-D size is four bits, two for each dimension. In general, the CIC-D size is  $2D$  bits for a  $D$ -dimensional system. For example, the CIC-D size for a 3-dimensional system is six bits.

Having defined the CIC-D encoding scheme, we may now define three different distance functions:

- the *geographical distance*,
- the *cell distance*, measured in cells,
- the *tile distance*, measured in tiles.

Since all the tiles within an LA are of equal size, the *tile distance* measured along either dimension is a linear function of the *geographical distance*, with a precision of one tile. However, the tile size can be adjusted to fit the size of a typical cell within the LA, and may differ from one LA to another. Hence, since our interest is in measuring/estimating the *cell distance*, the use of the *tile distance* is more effective for our purpose than the *geographical distance*. For example, a typical cell length in an urban area may be 10 times smaller than a typical cell length in a rural area. For this reason, the CIC-D encoding scheme is superior, in our opinion, to the use of GPS based distance tracking, both technically and economically.

**Lemma 1.** The *tile location* of the user (relative to a reference point), is uniquely determined by the list of the CIC-D

indices of all the *tiles* along its path, sorted in chronological order.

*Proof.* The proof follows from the fact that a modulo-3 increasing indexing uniquely identifies the direction of motion along each dimension. For example, if the index increases from west to east and the indices used are 0, 1, 2, then the transitions  $0 \rightarrow 1$ ,  $1 \rightarrow 2$ , and  $2 \rightarrow 0$  reflect eastward movements, while the transitions  $1 \rightarrow 0$ ,  $2 \rightarrow 1$ , and  $0 \rightarrow 2$  reflect westward movements. Thus, any change in the CIC-D indices has a unique interpretation of the user motion, and vice versa: any movement from a tile to any of its nearest neighbors is reflected by a unique change in the CIC-D indices. Consequently, the tile location of the user, relative to a reference point (e.g., the point of terminal power on), is uniquely determined by the list of all CIC-D indices along its path.  $\square$

Lemma 1 implies that if the cells are all equal size squares, then the *cell distance* traveled by the user can be computed via the CIC-D indices.

However, in reality cellular networks are not restricted to a certain topology. Moreover, the cells may differ in both shape and size. The purpose of the following analysis is to obtain the conditions on the system topology, under which the CIC-D encoding scheme guarantees a precise measurement of the *tile distance*. The coexistence of cells having different shapes and sizes within the same system implies that many cells may span on more than one tile. This creates a situation in which the CIC index detected by the user may differ from that of the tile at which the user resides. This situation is a potential source for an error that, under certain conditions, may cause a failure of the CIC-based method for distance measurement. It is interesting to note that the problem addressed here is somewhat similar to that faced by the verification of sliding window protocols, such as go-back-N [11]. In these protocols modulo counting of packets is proved to be a safe substitute to regular counting, despite the presence of line communication errors. The problem addressed here differs from that problem in that the modulo counting must be verified in  $D$  dimensions, and that the counters can either increase or decrease.

We next provide simple necessary and sufficient condition, under which the CIC-D encoding scheme guarantees a precise measurement of the tile location, relatively to a reference point (e.g., terminal power on). Hence, the tile distance traveled by the user can be computed, with a precision of one tile. Let  $l$  be the tile length of a uniform grid superposed on a two dimensional system. Let  $x$  be the maximal length of a cell within the system, in either dimension. Then, the following theorems hold:

**Theorem 2.** If  $x \leq l$ , the CIC-D encoding scheme guarantees a precise measurement of the user *tile location*, with a precision of one tile.

The proof is given in appendix A.

**Theorem 3.** If  $x > l$ , there exist topologies for which the CIC-D scheme fails in computing the tile location.

The proof is given in appendix B.

Our interest is in the *cell distance* traveled by the user. In order to convert the *tile distance* to the *cell distance*, some assumptions must be added about the system topology. Specifically, we assume that within the LA, the cell density per sufficiently small unit area is homogeneous. Let  $x$  be the maximal length of a cell within the LA, in either dimension. It is assumed that there exists a number  $\alpha \geq 1$ , such that the LA coverage area is significantly larger than  $\alpha x^2$ , and if we superpose a grid on the LA, with tile length equal to  $\sqrt{\alpha}x$ , then the number of cells associated with each tile (in the sense that they broadcast the same CIC-D index) is, for a good approximation, a constant  $C$ . We refer to this topology as the *generalized grid topology*.

**Definition.** The cells associated with a certain tile, in the sense that they broadcast the CIC-D index of that tile, are called the *associated cells* of that tile.

**Lemma 4.** In a generalized grid system, the *cell distance* traveled by the user can be determined by tracing the CIC-D indices, with a precision of  $C$  cells.

*Proof.* Choosing a tile length equal to  $\sqrt{\alpha}x$ , we get a grid topology system, in which  $x \leq l = \sqrt{\alpha}x$ , and each tile contains  $C$  cells. Using theorem 2, the CIC-D indices uniquely determine the tile distance. Hence, the *cell distance* traveled by the user is determined with a precision of  $C$  cells.  $\square$

Lemma 4 implies that in a *generalized grid* system, if all the cells *associated* with the same tile are paged simultaneously, the (cell) distance-based tracking strategy can be implemented using the CIC-D indexing.

*Remark 1.* Note that the *geographical distance* can be always derived from the *tile distance*, with a precision of one tile length.

The distance  $d$  between locations  $(x, y)$  and  $(x + \Delta x, y + \Delta y)$  can be derived by one of three alternative metrics:

- the *Euclidean* metric:  $d_E = \sqrt{\Delta x^2 + \Delta y^2}$ ,
- the *city block* metric (also known as the *Manhattan* metric):  $d_C = |\Delta x| + |\Delta y|$ ,
- the *maximum value* metric, also known as the *chess-board* metric:  $d_M = \max\{|\Delta x|, |\Delta y|\}$ .

Note that under all these metrics, the distance value is uniquely defined as a function of  $\Delta x$  and  $\Delta y$ .

*Remark 2.* Note that using hexagonal tiles, a modulo-7 indexing that uses only 3 bits (7 colors), is sufficient to implement a distance based strategy. The reason for this is that it is possible to color the hexagonal grid using seven colors such that

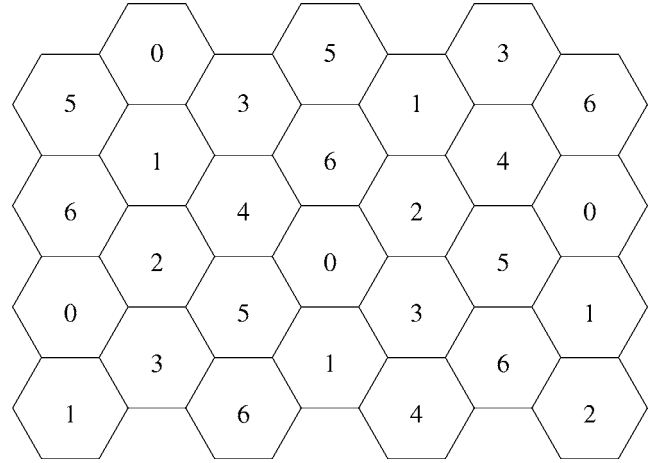


Figure 3. CIC-D encoding for distance-based strategy, using hexagonal tiles. A modulo-7 indexing, that uses only 3 bits (7 colors), is sufficient to implement a distance-based strategy.

each tile and the six tiles adjacent to it all get different colors. One way of doing that is shown in figure 3. The coloring is obtained in the following way. An arbitrary tile is given the color 0. Then, if a tile is given the color  $x$ , then the six tiles surrounding it, starting from the tile above it and going clockwise, are assigned the colors  $x + 6, x + 2, x + 3, x + 1, x + 5$  and  $x + 4$ , where the additions are performed modulo 7. It is not difficult to see that this consistency assigns colors to all the tiles, and that the coloring has the required property.

## 2.2. CIC encoding scheme for user movement tracking

In this section we consider the movement-based strategy, suggested in [3] and [1]. Under this strategy, the user counts the number of cell transitions, and transmits a registration message whenever this number exceeds a predefined threshold. In order to detect a cell boundary crossing, it is sufficient to assign each cell a different CIC from its nearest neighbors. Hence, this is a special case of the graph coloring problem. Given a wireless networks, its adjacency graph is defined as an undirected graph in which each cell is represented by a vertex, and there is an edge connecting two vertices if and only if these vertices represent neighboring cells. The problem of implementing the movement-based strategy using CIC is therefore the problem of finding the smallest number of colors needed such that every vertex in the equivalent adjacency graph is assigned one color, and no two vertices connected by an edge get the same color. For a general planar graph at most four colors are required for such a coloring. Hence, for a wireless network whose adjacency graph is a planar graph, at most four different code words are required to implement the movement-based strategy. For simpler topologies even fewer codes may suffice: for example, only three colors are required for hexagonal shaped cells. Figure 4 depicts the movement-based CIC encoding for a hexagonal grid system.

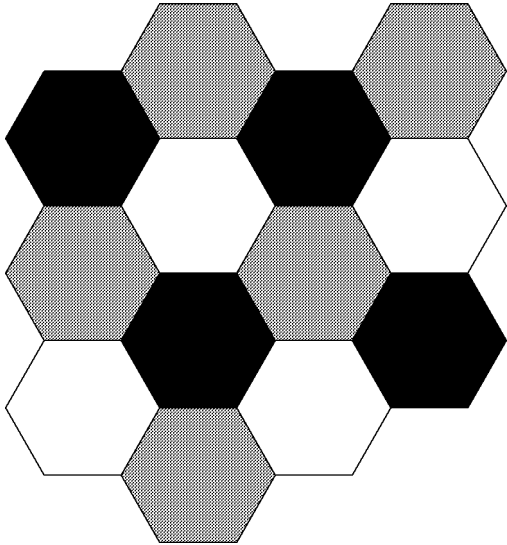


Figure 4. Movement-based CIC encoding for hexagonal shaped cells.

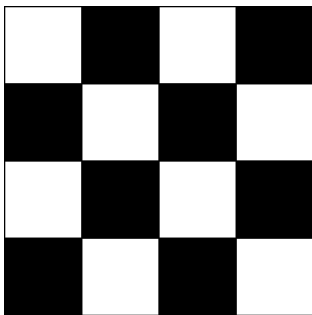


Figure 5. Movement-based CIC encoding for square shaped cells. Two codes are sufficient for user movement along either vertical or horizontal direction.

For square shaped cells, if only cells having a common border are defined as neighboring cells, where a cell boundary crossing is allowed only in horizontal or vertical direction, then only two codes are required. Figure 5 depicts the CIC encoding for this system.

Note that an alternative approach for detecting a cell boundary crossing is the identification of a drop in the radio signal power. However, such a drop can often occur inside a cell, for example, as a result of radio propagation interference. Using a CIC for detecting a cell boundary crossing is therefore more reliable.

### 2.3. The conditional timer strategy

In [10] a *timer-based* method was suggested in which the user updates its location every  $T$  time units, where  $T$  is a parameter. Each time the user makes no contact with the network for  $T$  units of time – it initiates a registration message. The timer-based method is considered as the simplest to implement tracking strategy, because the user is not required to process any location information. However, as shown below, the use of location information can significantly improve the performance of the timer-based method. To illustrate this idea, consider, for example, the following condi-

tional timer (CT) method. The user transmits a registration message every  $T$  time units, *provided* that its present location differs from its last known location. Clearly, the paging cost under the CT strategy is identical to that of the simple timer-based method. However, the registration cost of the CT method is lower than that of the timer-based method, because unnecessary registration messages are avoided. Hence, the CT strategy outperforms the timer-based method whenever the expected reduction in registration cost exceeds the CIC transmission cost. Note that the CT strategy is independent of the system topology.

To compare the performance of the CT strategy to that of the timer-based strategy, consider an LA consisting of  $N$  cells and  $n$  users, and let  $T$  be the time threshold used. Under the CT strategy, the user examines its current location every  $T$  time units, and if the new location differs from the last known location – the user transmits a registration message. Let  $l(t)$  be the user location at time  $t$ . If the last registration message was transmitted at time  $t = 0$ , then the condition for the next registration message is that  $l(T) \neq l(0)$ . Assuming that user motion is Markovian and location independent, let  $P(T)$  be the probability that the user will remain at its location after time  $t = T$ , namely,  $P(T) = \Pr[l(t - T) = l(t)]$ . Let  $l_r$  be the cost of registration, measured in the number of bits. Comparing to the timer-based method, the registration cost rate under the CT method is reduced by

$$\Delta_r = nl_r \left[ \frac{1}{T} - \frac{1 - P(T)}{T} \right] = \frac{nP(T)l_r}{T}. \quad (1)$$

Let  $T_1$  be the time period between two successive CIC messages under the CT strategy, CIC-CT in short. Since a CIC-CT message uniquely identifies each cell, the length of the message must be equal to  $\log_2 N$ , and the CIC transmission cost rate is given by

$$T_{CT} = \frac{N \log_2 N}{T_1}. \quad (2)$$

The condition under which the CT strategy outperforms the timer strategy is  $\Delta_r > T_{CT}$ , implying that

$$P(T) > \frac{\log_2 N}{l_r} \left( \frac{T}{T_1} \right) \frac{1}{\rho}, \quad (3)$$

where  $\rho = n/N$  is the number of users per cell. For example, in the GSM system the minimal cost of registration is 55 bytes [4]. Hence, substituting in equation (3)  $\rho = 1200$  users per cell,  $N = 128$  cells,  $T = 10$  min, and  $T_1 = 1$  s, we get that for  $P(T) > 0.008$  the CT strategy outperforms the timer-based strategy. Equation (3) implies that this superiority increases with the user density  $\rho$ .

The implementation of the CT strategy is very simple, and independent of the system topology: a unique code is assigned to each cell. The CIC-CT indexing also supports location-dependent tracking strategies. For example, each user can store in its memory the CIC of its home, office, and other locations.

#### 2.4. The timer/movement (TM) strategy – a combination of a timer with a movement counter

The TM strategy combines a timer with a movement counter as follows:

- The user counts the number of cell transitions since its last location update.
- Every  $T$  time units the user checks the number of cell transitions  $k$  since its last location update.
- If  $k$  exceeds a predefined threshold  $M$ , the user transmits a registration message and the movement counter is set to zero.
- Otherwise, the user does not register. Future cell transitions will accumulate at  $k$ , and the movement counter will be checked again after additional  $T$  time units.

Note that this strategy guarantees the registration rate to be bounded from above by  $1/T$ . The movement-based strategy is a special case of the TM strategy, with a timer equal to one time unit.

### 3. Performance analysis

The purpose of this section is to compare the CIC-based strategies and the tracking strategies used by existing systems, and to show the difference of each strategy. Existing cellular networks use the geographic-based (GB) strategy. In section 3.1 we show that the CIC-based strategies can only perform better than the GB strategy. Then, we compare between the CIC-based strategies and the timer-based method, as adopted by many systems who incorporate the IS-41 standard with a fixed (static) timer, and show the difference of each strategy.

To model user movement in the network we assume that time is slotted and that the user can make at most one move during a single time slot. The movements are stochastic and independent from one user to another. We assume that calls are initiated by the users as a Poisson process, and that the size of a time slot is sufficiently small such that we can neglect the probability of arrival of more than one call during a single time slot. For each user, the probability of a call arrival during each time slot is denoted by  $\lambda$ . We consider an infinite two-dimensional system with grid topology. The user motion model is assumed to be independent and identically distributed (i.i.d. model): a user residing at cell  $(i, j)$  can move to cells  $(i + 1, j)$ ,  $(i - 1, j)$ ,  $(i, j + 1)$ , and  $(i, j - 1)$  with probability  $p$  to each one of them, or to remain at cell  $(i, j)$  with probability  $q = (1 - 4p)$ . The distance  $d$  between two cells, in terms of number of cells, is measured using the *city block metric*, also known as the *Manhattan metric*:

$$\text{distance}((i_1, j_1), (i_2, j_2)) = |i_1 - i_2| + |j_1 - j_2|. \quad (4)$$

The paging strategy is the following. The user is first searched at its last known location, say  $(x, y)$ . If it is not found there, then it is searched sequentially in increasing distance order

from  $(x, y)$ . The search is conducted simultaneously at all the cells in a distance  $d$  from  $(x, y)$ ,  $d = 1, 2, 3, \dots$ , until the user is found.

The comparison between the different strategies is done by evaluating the expected paging cost, denoted  $P_s$ , the expected registration cost, denoted  $R_s$ , and the CIC transmission cost, denoted  $T_s$ , where  $s$  is the strategy used. The tracking cost is given by  $P_s + R_s + T_s$ . Since user movements are stochastic and independent from one user to another, it suffices to calculate the tracking cost for one user (arbitrarily chosen). We compare between the distance-based strategy, the movement-based strategy, the TM strategy, and the timer-based method (as implemented in practice, using a fixed timer).

#### 3.1. The geographic-based (GB) strategy

The goal of the CIC concept is not to replace the GB strategy, but rather to improve its performance, by adapting the registration activity *within* the LA to the individual user parameters. Since the GB strategy requires the user to register only at the border between different LAs, while within the LA the user never registers, there is a significant amount of non-utilized system resources *within* the LA, that can be used for dynamic user registration. Hence, using a CIC-based strategy in *addition to* the GB strategy would certainly decrease the paging cost, with only a moderate increase in the registration cost.

Consider the following CIC-based strategy. Each cell transmits the LA ID currently transmitted in existing networks, in addition to its CIC. The conclusion from the above discussion is that this strategy, that incorporates the GB method with a CIC-based strategy, can only perform better than the GB strategy. This is since the paging cost associated with this strategy is lower than that of the GB strategy, while the registration rate can always be adjusted to the system load [8] such that only non-utilized system resources are used.

#### 3.2. Performance of the distance-based strategy

Let  $(x_0, y_0)$  be the user last known location, at time  $t = 0$ . Using the *Manhattan metric*, the group of all locations at a distance less than  $D$  is the collection of all locations  $(x, y)$  satisfying  $|x - x_0| + |y - y_0| < D$ . These locations form a rhombus centered at  $(x_0, y_0)$ , with side length  $\sqrt{2}D$ . The *layer*  $l(d)$  ( $d > 0$ ) is defined as the group of all locations at a distance  $d$  from  $(x_0, y_0)$ . There are exactly  $4d$  locations in  $l(d)$ . Hence, the maximal number of all possible locations is given by

$$1 + \sum_{d=1}^{D-1} 4d = 1 + 2D(D-1). \quad (5)$$

Let  $d_x, d_y$  be the distance traveled from the user last known location along the horizontal and vertical direction, respectively. The distance  $d$  from the user last known location is given by  $d = d_x + d_y$ . Each layer  $l(d)$  is partitioned into two

disjoint groups: the group  $l_1(d)$  is the collection off all locations satisfying  $d_x + d_y = d$ ,  $d_x \neq 0$ ,  $d_y \neq 0$ , while the group  $l_2(d)$  is the collection of all locations satisfying  $d_x + d_y = d$ , where either  $d_x = 0$  or  $d_y = 0$ . Note that for  $1 \leq d \leq D-1$ , there are exactly 4 locations in  $l_2(d)$ , and  $4(d-1)$  locations in  $l_1(d)$ . Since  $\lambda$  is the probability of a call arrival, a user located at  $l(d)$  can move to the layer  $l(0)$  with probability  $\lambda$ . A user located at  $l_1(d)$  ( $0 < d < D-1$ ) can either move to the layer  $l(d-1)$  with probability  $(1-\lambda)2p$ , move to the layer  $l(d+1)$  with probability  $(1-\lambda)2p$ , or remain at its current location with probability  $(1-\lambda)(1-4p)$ . A user located at  $l_2(d)$  ( $0 < d < D-1$ ) can either move to the layer  $l(d-1)$  with probability  $(1-\lambda)p$ , move to the layer  $l(d+1)$  with probability  $(1-\lambda)3p$ , or remain at its current location with probability  $(1-\lambda)(1-4p)$ . The distance traveled by the user from its last known location is described by a Markov chain, in which the state is defined by  $(d_x, d_y)$ , where  $d_x + d_y < D$ . Whenever  $d_x + d_y = D$  the user transmits a registration message and immediately reaches the state  $(0, 0)$ .

Let  $S(d_x, d_y)$  denote the steady state probability to find the user at state  $(d_x, d_y)$ . The equilibrium equations for the Markov chain are given by

$$\begin{aligned} & [4p(1-\lambda) + \lambda]S(0, 0) \\ &= \lambda + p(1-\lambda) \left[ \sum_{l(1)} S(d_x, d_y) + 2 \sum_{l_1(D-1)} S(d_x, d_y) \right. \\ & \quad \left. + 3 \sum_{l_2(D-1)} S(d_x, d_y) \right]. \end{aligned} \quad (6)$$

For  $(d_x, d_y) \in l_1(d)$ ,  $D-1 > d > 0$ :

$$[4p(1-\lambda) + \lambda]S(d_x, d_y) = (1-\lambda)pS'(d_x, d_y), \quad (7)$$

where  $S'(d_x, d_y) = S(d_x-1, d_y) + S(d_x+1, d_y) + S(d_x, d_y-1) + S(d_x, d_y+1)$ .

For  $(d_x, d_y) \in l_2(d)$ ,  $D-1 > d > 0$ :

$$[4p(1-\lambda) + \lambda]S(d_x, d_y) = (1-\lambda)pS''(d_x, d_y), \quad (8)$$

where  $S''(0, d_y) = S(0, d_y+1) + S(0, d_y-1) + 2S(1, d_y)$ , and similarly, for  $d_y = 0$ :  $S''(d_x, 0) = S(d_x-1, 0) + S(d_x+1, 0) + 2S(d_x, 1)$ .

For  $d = D-1$ ,  $(d_x, d_y) \in l_1(D-1)$ :

$$\begin{aligned} & [4p(1-\lambda) + \lambda]S(d_x, d_y) \\ &= (1-\lambda)p[S(d_x-1, d_y) + S(d_x, d_y-1)]. \end{aligned} \quad (9)$$

For  $d = D-1$ ,  $(d_x, d_y) \in l_2(D-1)$ :

$$[4p(1-\lambda) + \lambda]S(0, D-1) = (1-\lambda)pS(0, D-2), \quad (10)$$

$$[4p(1-\lambda) + \lambda]S(D-1, 0) = (1-\lambda)pS(D-2, 0). \quad (11)$$

The steady state probabilities can be obtained using standard numerical procedures for solving equations (6)–(11). Figure 6 depicts the steady state probabilities for  $D = 10$  and  $D = 20$ , as computed numerically for  $\lambda = 0$ .

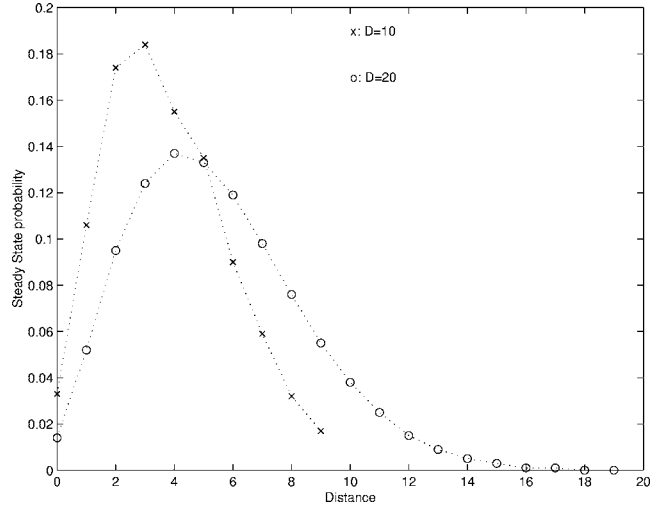


Figure 6. Distance-based strategy: steady state probabilities for  $\lambda = 0$ .

Let  $\pi_d$  denote the probability that under steady state the user is at a distance  $d$  from its last known location. The expected paging cost is given by

$$\begin{aligned} P_D &= \sum_{d=0}^{D-1} [1 + 2d(d+1)]\pi_d l_p \\ &= \left[ 1 + \sum_{d=1}^{D-1} 2d(d+1)\pi_d \right] l_p, \end{aligned} \quad (12)$$

where  $l_p$  is the cost of a single paging, measured in number of bits. A registration message is transmitted whenever a user located in a distance  $D-1$  from its last known location moves to location in a distance  $D$  from its last known location. That happens with probability  $2p$  if the user location belongs to  $l_1(D-1)$ , and with probability  $3p$  if it belongs to  $l_2(D-1)$ . Let  $\pi_d^1$ ,  $\pi_d^2$  be the steady state probabilities to be in  $l_1(d)$ ,  $l_2(d)$ , respectively. The expected registration cost is, therefore,

$$R_D = [2p\pi_{D-1}^1 + 3p\pi_{D-1}^2]l_r, \quad (13)$$

where  $l_r$  is the cost of a single registration, measured in number of bits. For  $D \gg 1$ ,  $\pi_{D-1}^2 \ll \pi_{D-1}^1$ , and we can approximate  $R_D \approx 2p\pi_{D-1}^1 l_r$ . The CIC transmission cost is four bits transmitted by each cell, every  $T_1$  time units. The cost per user is, therefore,

$$T_D = \frac{4}{T_1 \rho}, \quad (14)$$

where  $\rho = n/N$  is the number of users per cell.

### 3.3. Performance of the movement-based strategy

Since user movements are allowed only in horizontal or vertical direction, only one bit is required to detect a cell boundary crossing. Hence, the CIC transmission cost (per user) is given by

$$T_M = \frac{1}{T_1 \rho}. \quad (15)$$

Since the probability of cell boundary crossing is  $4p$  for each time slot, it takes on average  $M/(4p)$  time slots to complete  $M$  movements. The expected registration cost is, therefore,

$$R_M = \left[ \frac{4p}{M} \right] l_r. \quad (16)$$

Since calls are initiated by the users as a Poisson process, the PASTA (Poisson Arrival See Time Average) property implies that the probability of a paging event given that the user made  $k$  movements since its last registration message is uniformly distributed over  $[0, M - 1]$ . The expected paging cost is, therefore, given by

$$P_M = \frac{l_p}{M} \sum_{k=0}^{M-1} \sum_{d=0}^k \Pr[\text{distance} = d \mid \text{movements} = k] \times [1 + 2d(d+1)]. \quad (17)$$

In order to obtain the value of  $\Pr[\text{distance} = d \mid \text{movements} = k]$  we first consider the one-dimensional case: given that the user made  $k$  movements, the probability that the distance traveled is equal to  $d$  is given by the sum of all probabilities to have  $d + (k-d)/2$  movements in one direction, and  $(k-d)/2$  movements along the opposite direction. Hence, for the one-dimensional case we get

$$\Pr[\text{distance} = d \mid \text{movements} = k] = \frac{1}{2^k} \begin{cases} 2 \binom{k}{(k-d)/2} & \text{for } 0 < d \leq k, k-d \text{ even,} \\ \binom{k}{k/2} & \text{for } d = 0, k \text{ even,} \\ 0 & \text{otherwise.} \end{cases} \quad (18)$$

The factor 2 in equation (18) results from the fact that for  $d > 0$  the user can move either to the right or to the left. For the two-dimensional case, given that the user made  $k$  movements, the probability of traveling a distance  $d$  is the sum of all probabilities of traveling a distance  $d_x$  along the horizontal axis ( $0 \leq d_x \leq d$ ) by making  $k_x$  movements along the horizontal axis ( $d_x \leq k_x \leq k$ ) and travel a distance  $d_y = d - d_x$  by making  $k - k_x$  movements along the vertical axis. Note that  $d_y$  must satisfy  $d_y \leq k_y = k - k_x$ . Given  $d_x, k_x, k, d$ , the number of all possible paths is

$$\alpha(k_x, d_x, k, d) = \begin{cases} \delta_x \delta_y \binom{k_x}{(k_x - d_x)/2} \binom{k - k_x}{[(k - k_x) - (d - d_x)]/2} & \text{for } k_x - d_x \text{ even, } k - d \text{ even,} \\ 0 & \text{otherwise,} \end{cases} \quad (19)$$

where  $\delta_x, \delta_y = 2$  if  $d_x, d_y > 0$ , respectively, and  $\delta_x, \delta_y = 1$  if  $d_x, d_y = 0$ , respectively. Hence,

$$\Pr[\text{distance} = d \mid \text{movements} = k] = \frac{1}{4^k} \sum_{k_x=0}^k \binom{k}{k_x} \sum_{d_x=\max\{0, d-(k-k_x)\}}^{\min\{k_x, d\}} \alpha(k_x, d_x, k, d). \quad (20)$$

The constraints on the summation in equation (20) are due to the following conditions:  $d_x \leq k_x \leq k$ ,  $d_x \leq d$ , and

$d - d_x = d_y \leq k_y = k - k_x$ , which implies that  $d_x \geq d - (k - k_x)$ . Substituting equation (20) in equation (17), we get the expected paging cost under the movement based strategy.

#### 3.4. Performance of the timer-based strategy

The timer-based strategy requires no location information. Hence, its CIC transmission cost is zero. The registration cost is simply

$$R_T = \frac{l_r}{T}, \quad (21)$$

where  $T$  is the timer used. Using a similar reasoning to that used for the movements-based method, the probability of a paging event  $t$  time units after the user last registration message is uniformly distributed over  $[0, T - 1]$ . The expected paging cost is, therefore,

$$P_T = \frac{l_p}{T} \sum_{t=0}^{T-1} \sum_{k=0}^t \Pr[\text{movements} = k \mid \text{time} = t] \times \sum_{d=0}^k \Pr[\text{distance} = d \mid \text{movements} = k] \times [1 + 2d(d+1)]. \quad (22)$$

Since the last term is given in equation (20), what remains to be computed is the probability that the user had made  $k$  movements during  $t$  time units. Using that the user can make at most one movement during one time slot, we get that

$$\Pr[\text{movements} = k \mid \text{time} = t] = \begin{cases} \binom{t}{k} (4p)^k (1 - 4p)^{t-k} & \text{for } 0 \leq k \leq t, \\ 0 & \text{for } t < k. \end{cases} \quad (23)$$

Substituting equations (23) and (20) in equation (22) we get the expected paging cost.

#### 3.5. The timer/movement (TM) strategy

Under the TM strategy the user counts the number of cell transmissions. The CIC transmission cost is therefore equal to that of the movement-based strategy:

$$T_{TM} = \frac{1}{T_1 \rho}. \quad (24)$$

Consider the special case of the TM method where  $k = 0$ . Under this strategy, denoted  $TM_0$ , the user transmits a registration message every  $T$  time units, provided it had made at least one movement since its last location update. Hence, the registration rate is given by

$$R_{TM_0} = l_r \left[ \frac{1 - q^T}{T} \right], \quad (25)$$

where  $q = 1 - 4p$  is the probability that the user has not moved during a single time slot. Comparing to the timer-



based method, the registration rate reduces, since users who have not moved do not register:

$$R_T - R_{TM_0} = l_r \left[ \frac{q^T}{T} \right]. \quad (26)$$

Since under both strategies a user who has changed its location since its last location update transmits a registration message, the paging cost under both strategies is the same:

$$P_{TM_0} = P_T. \quad (27)$$

Hence, equations (24), (26) and (27) imply that the condition under which the  $TM_0$  strategy outperforms the timer based strategy is

$$q^T > \frac{T}{T_1 l_r \rho}. \quad (28)$$

Consider, for example, the numerical example given in section 2.3, where  $\rho = 1200$  users per cell,  $l_r = 440$  bits,  $T = 10$  min and  $T_1 = 1$  s. For this example we get that the  $TM_0$  strategy outperforms the timer-based strategy whenever  $q^T > 0.0011$ , namely, if at least 0.11% of the users do not move from their location during 10 min. Over this range of parameters, the  $TM_0$  strategy outperforms the timer-based method in the sense that for the same expected paging cost, the sum of the CIC transmission cost and the expected registration cost is smaller. Equation (28) implies that this superiority increases with the user density  $\rho$ , and decreases with the timer  $T$ .

### 3.5.1. A comparison of the TM strategy to the conditional timer (CT) strategy

Under both strategies a user who has changed its location during  $T$  time units since its last location update transmits a registration message. Hence, the paging cost under both strategies is the same. The registration rate under the  $TM_0$  strategy is higher than that of the CT strategy, since there is a probability that during  $T$  time units the user has moved in and out from its last known location. In such a case, the registration message transmitted by the user is unnecessary, since the user location has not changed since its last location update. Let  $\beta(T)$  denote the user probability to move out from its last known location and return back during  $T$  time units:  $\beta(T) = \Pr[\text{movements} > 0 \mid \text{time} = T] \cdot \Pr[\text{distance} = 0 \mid \text{movements} > 0]$ . The difference in the registration rate is given by

$$\Delta_r = R_{TM_0} - R_{CT} = \frac{\beta(T) l_r}{T}. \quad (29)$$

On the other hand, the CIC transmission cost of the  $TM_0$  strategy is much lower than that of the CT strategy:

$$\Delta_T = T_{CT} - T_{TM_0} = \frac{\log_2 N - 1}{T_1 \rho} = \frac{\log_2 [N/2]}{T_1 \rho}. \quad (30)$$

Note that both  $\Delta_r$  and  $\Delta_T$  are measured in terms of number of bits per time unit. The condition under which the  $TM_0$  strat-

egy outperforms the CT strategy is that  $\Delta_r < \Delta_T$ , implying that

$$\beta(T) < \frac{\log_2 [N/2]}{l_r \rho} \frac{T}{T_1}. \quad (31)$$

Equation (31) implies that the efficiency of the  $TM_0$  strategy in comparison to the CT strategy decreases with the user density  $\rho$  and the cost of a single registration  $l_r$ , and increases with  $\log_2 N$  and the CIC transmission rate  $1/T_1$ .

The probability  $\beta(T)$  is equal to the sum of all probabilities of traveling a distance equal to zero *while* performing  $m$  movements, where  $m > 0$ . Since the distance traveled by the user is zero, the number of movements the user made along each direction must be equal to the number of movements along the opposite direction. Hence, the number of movements  $m$  must be an even number. Using equations (23), and applying equation (18) on both dimensions, we get that

$$\begin{aligned} \beta(T) &= \sum_{k=1}^{\lfloor T/2 \rfloor} \Pr[\text{movements} = 2k \mid \text{time} = T] \\ &\quad \times \Pr[\text{distance} = 0 \mid \text{movements} = 2k] \\ &= \sum_{k=1}^{\lfloor T/2 \rfloor} \binom{T}{2k} (4p)^{2k} (1-4p)^{T-2k} \\ &\quad \times \frac{1}{4^{2k}} \sum_{i=0}^k \binom{2i}{i} \binom{2(k-i)}{k-i}. \end{aligned} \quad (32)$$

The explanation for equation (32) is as follows.  $\beta(T)$  is the sum of all probabilities to travel a zero distance in  $2k$  movements, where  $0 < 2k \leq T$ .  $2i$  movements ( $0 \leq 2i \leq 2k$ ) are along the horizontal direction, where  $i$  movements are to the right and  $i$  movements are to the left. The other remaining  $2k - 2i$  movements are along the vertical direction, from which  $k - i$  movements are upward and  $k - i$  movements are downward.

Substituting  $\beta(T)$  in equation (31) we get the range of the parameter  $p$  over which the  $TM_0$  strategy outperforms the CT strategy, and vice versa. For example, substituting  $N = 128$  cells,  $\rho = 1200$  users per cell,  $l_r = 440$  bits,  $T = 10$  min, and  $T_1 = 1$  s in equation (31), we get that the  $TM_0$  strategy outperforms the CT strategy whenever the user probability to move out from its last known location and return back during 10 min is less than  $3/440 \approx 0.07$ . Using equation (32), we get that  $\beta(T) < 0.007 < 0.07$  over all possible values of  $p$  ( $0 \leq p \leq 0.25$ ). Hence, for this numerical example, the  $TM_0$  strategy is *always* superior to the CT strategy.

## 4. Comparison of the strategies and numerical results

To compare the performance of the tracking strategies discussed we evaluate the tracking cost under two search strategies, representing two extreme paging delay constraints:

- The sequential search described in section 3. This search is efficient in the sense that the average number of locations searched is relatively small. However, the paging delay is relatively large.

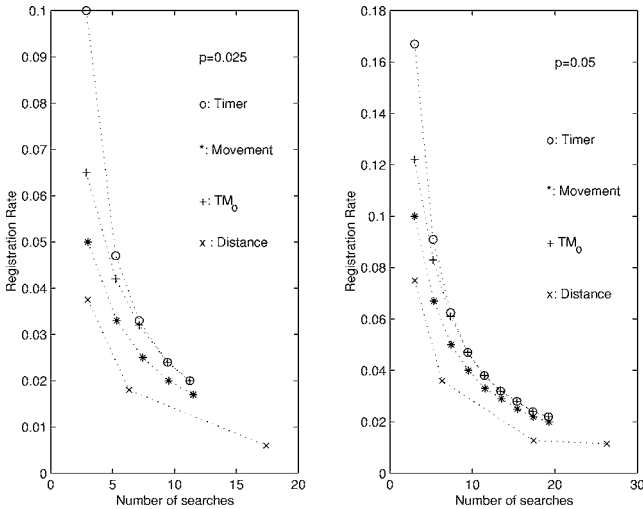


Figure 7. Registration rate as a function of the expected number of searches, under sequential paging.

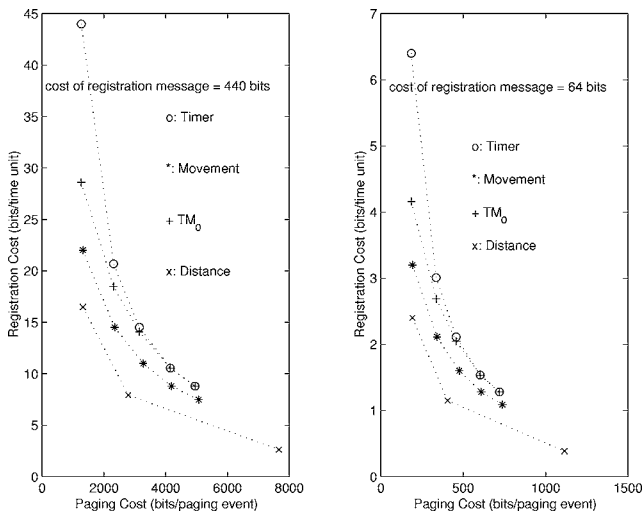


Figure 8. The total registration cost as a function of the paging cost, under sequential paging, for CIC transmission rate of 5 messages per time unit.

- A one-phase paging. Under this paging strategy the search is conducted simultaneously at all possible candidates for the user location.

Figure 7 depicts the expected registration rate (measured in number of registrations per time unit) as a function of the expected number of searches for the timer-based, movement-based,  $TM_0$ , and the distance-based strategies, under sequential paging, for  $p = 0.025$  and  $p = 0.05$ . It can be seen that all CIC-based strategies outperform the timer-based method, in the sense that for the same expected number of searches required to find the user, the expected registration rate is significantly smaller than the one required by the timer-based method. The best performance is achieved by the distance-based strategy. Note that for very low registration rates (implying high paging cost) the performance difference between the movement-based, timer-based, and the  $TM_0$  strategies is negligible. This result is in agreement with the result obtained in [3] for a one-dimensional model.

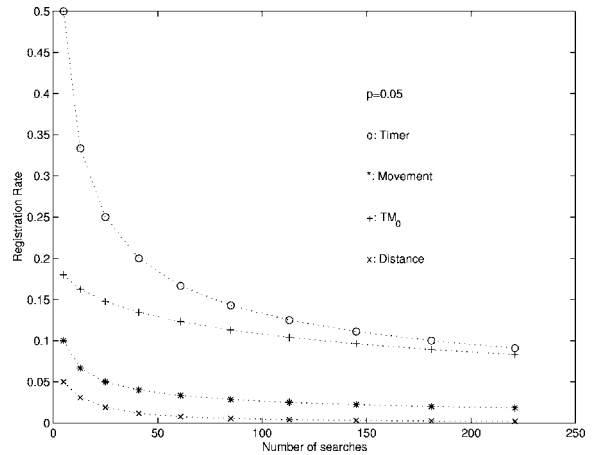


Figure 9. Registration rate as a function of the number of searches, under single phase search, for  $p = 0.05$ .

To compare the *overall* performance of the tracking strategies, the CIC transmission cost must be taken into account. Figure 8 depicts the *total* registration cost, defined as the sum of the registration cost and the CIC transmission cost, as a function of the paging cost, for CIC transmission rate of 5 messages per time unit,  $p = 0.025$ , and user density  $\rho = 1200$  users per cell. The cost of a single paging is considered equal to the cost of a single registration message,  $l_p = l_r = 440$  bits and  $l_p = l_r = 64$  bits. The CIC-based strategies outperform the timer-based strategy. The best performance is achieved by the distance-based strategy, even though its CIC transmission cost is the maximal. Note that the minimal cost of registration in the GSM system is 440 bits [4], for the situation where the VLR at the time of registration is the last known VLR.

Figure 9 depicts the registration rate as a function of the expected number of searches, for one-phase paging, subject to the constraint that the maximal paging delay is one. It can be seen that the CIC-based strategies outperform the timer-based method even better than under sequential paging.

Equations (14), (15), (24), and (28) imply that the CIC-based strategies superiority over the timer-based method increases with the user density  $\rho$ . The reason for this is that the CIC transmission cost (per user) decreases with  $\rho$ , while the expected reduction in the registration rate and paging cost of a single user is independent of  $\rho$ .

## 5. Summary and concluding remarks

We presented a novel approach for user tracking, based on broadcasting of cell identification code. We propose several cell identification codes (CIC) encoding schemes, which are used to implement different tracking schemes: the movement-based, the TM, the conditional timer (CT), and the distance-based strategy. We showed that the CIC-based strategies outperform the timer-based method over a wide range of parameters. The best performance is achieved by a four-bit CIC, used to implement a distance-based tracking strategy, in a two-dimensional system. This strategy is optimal for the upcoming satellite-based wireless networks,

consisting of grid topology, and applicable on existing typical cellular networks. From an implementation point of view, the TM and the movement-based strategies are much simpler. Moreover, they are applicable on a wider range of system topologies. The implementation of the CT strategy is the most expensive, in terms of CIC length. However, this strategy is independent of the system topology, and its implementation is the simplest. The main advantage of the CIC-CT indexing is its capability to support location-dependent tracking strategies. For example, each user can store in its memory the CIC of its home, office, and other locations.

The main advantage of the CIC concept is in its practicality. A short message of one, and even four bits can be either condensed into the location area ID currently transmitted by each cell in the GSM system and in North America, or transmitted as a special broadcast message. In both cases the CIC transmission cost is negligible. Naturally, an implementation of a CIC-based strategy requires software modification of user equipment and base stations, but this is expected from any new tracking strategy. The CIC-based strategies proposed in this paper do not require significant computational power neither from the user, nor from the base station. Therefore, they can be easily implemented on existing hardware.

A combined strategy, that incorporates the GB method, used by current systems, with a CIC-based strategy, can only perform better than the GB strategy. All CIC-based strategies outperform the timer-based method, as adopted by many systems, over a wide range of parameters. Moreover, this superiority increases with the user density. The reason for this is that the cost associated with the need to transmit a CIC message depends only on the number of cells. Hence, the CIC transmission cost *per user* decreases with user density. On the other hand, the expected reduction in registration rate and paging cost of a single user is independent of the number of users per cell.

## Appendix A. Proof of theorem 2

Without loss of generality, it is sufficient to prove the theorem for one dimension, independently of the other dimensions. Consider for example the west-east dimension, where the index increases (modulo-3) from west to east. Since  $x \leq l$ , each cell spans on at most two bands. If a cell spans on two bands, its CIC-D index is the index of the easternmost band on which it spans. If the cell border is on the border between bands, it transmits the code of the band on which it spans. Hence, the CIC-D index detected by any point within the system coverage area may differ from the CIC-D index of that point by at most 1.

Below we show that this difference is not accumulated upon movement, and remains at most one. Hence, detecting the CIC-D indices along any path, determine the location of each point along that path, with a precision of one band.

*Proof.* Let  $d(p)$ , where  $p$  is the user location, be a counter maintained by the user as follows. Every modulo-3 increment in the CIC-D index detected by the user, increases  $d(p)$  by

one, while every modulo-3 decrement in the the CIC-D index detected by the user, decreases  $d(p)$  by one. Let  $d'(p)$  denote the real band location of the point  $p$ , relatively to the same reference point used by the user. The *location error*  $\Delta(p)$  is defined by  $\Delta(p) = d(p) - d'(p)$ . The *state* of a point  $p$ , denoted by  $S(p)$ , is defined as the modulo-3 difference between the CIC index detected at  $p$ , and the CIC index of the band containing  $p$ :  $S(p) = \text{CIC}_{\text{cell}} - \text{CIC}_{\text{band}}$ . For every point  $p$ , either  $S(p) = 0$ , or  $S(p) = 1$ . The state  $S(p) = 1$  reflects a situation where  $p$  is within a cell which spans on two bands, and transmits the index of the eastern neighboring band of  $p$ . The theorem follows directly from the following claim:

**Claim.**  $\Delta(p) = S(p), \forall p$ .

*Proof of the claim.* We distinguish between two situations. First, a movement within a cell. In this situation  $d(p)$  remains unchanged, while  $d'(p)$  changes only upon band crossing. Hence,  $\Delta(p)$  and  $S(p)$  change simultaneously. Moreover, they both either increase by one, or decrease by one. Consequently, the value of  $\Delta(p) - S(p)$  is invariant under a movement within a cell. The second situation is a cell boundary crossing. The proof of the claim for this situation is based on an induction on the number of cell transitions along the user path.

First, after 0 cell transitions we get that  $d(p) = 0$ , implying that  $\Delta(p) = -d'(p)$ . Using the same reasoning used for a movement within a cell, we get that  $d'(p) = -S(p)$ . Hence,  $\Delta(p) = S(p)$ . Given that after  $n$  cell transitions  $\Delta(p) = S(p)$ , we show that also for  $n + 1$  cell transitions  $\Delta(p) = S(p)$ . Let  $p_1, p_2$  be two points on the border between cells  $a$  and  $b$ , such that  $p_1 \in a, p_2 \in b$ . There are exactly four situations of movement from  $p_1$  to  $p_2$ :

1.  $S(p_1) = S(p_2) = 0$ . From the induction assumption,  $\Delta(p_1) = S(p_1) = 0$ . If  $p_1$  and  $p_2$  are within the same band, the conditions  $S(p_2) = 0$  and  $\Delta(p_1) = 0$  imply that  $\Delta(p_2) = 0 = S(p_2)$  (see figure 10, the transition from  $a_1$  to  $a_2$  and vice versa). If  $p_1$  and  $p_2$  are in different bands (the cell transition is coupled with a band transition), the conditions  $S(p_1) = 0$  and  $S(p_2) = 0$  imply that the movement  $p_1 \rightarrow p_2$  is interpreted correctly by the CIC indices (see the discussion in lemma 1). Therefore, the condition  $\Delta(p_1) = S(p_1) = 0$  implies that  $d'(p_2) = d(p_2)$ . Hence,  $\Delta(p_2) = 0 = S(p_2)$ .

2.  $S(p_1) = 0, S(p_2) = 1$ . The conditions  $S(p_2) = 1$  and  $x \leq l$  imply that  $p_2$  is not on the border between bands, otherwise the cell which contains  $p_2$  must be longer than  $l$ . Hence,  $p_1$  and  $p_2$  must be within the same band, implying that  $d'(p_2) = d'(p_1)$ . Since  $\Delta(p_1) = S(p_1) = 0$ , we get that  $\Delta(p_2) = 1 = S(p_2)$  (see figure 10, the transitions  $a_2 \rightarrow a_3$  and  $a_3 \rightarrow a_4$ ).

3.  $S(p_1) = 1, S(p_2) = 0$ . Since  $S(p_1) = 1$ , the condition  $x \leq l$  implies that  $p_1$  is not on the border between bands. Hence,  $p_1$  and  $p_2$  are within the same band, implying that

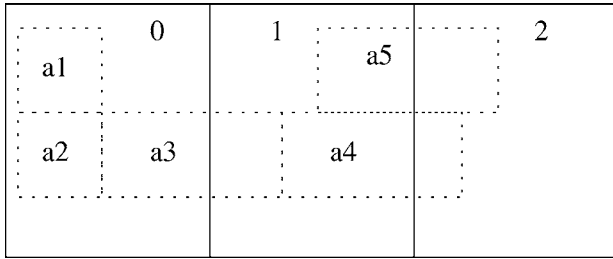


Figure 10. Cells  $a1$  and  $a2$  transmit the index 0,  $a3$  transmits the index 1, and cells  $a4$  and  $a5$  transmit the index 2.

$d'(p1) = d'(p2)$  and  $d(p1) = d(p2) + 1$ . Hence,  $\Delta(p2) = d(p2) - d'(p2) = \Delta(p1) - 1 = 0 = S(p2)$  (see figure 10, the transitions  $a3 \rightarrow a2$  and  $a4 \rightarrow a3$ ).

4.  $S(p1) = S(p2) = 1$ . The condition  $x \leq l$  implies that both  $p1$  and  $p2$  are not on the border between bands. Hence, they are within the same band, implying that  $d'(p1) = d'(p2)$  and  $d(p1) = d(p2)$ . Consequently,  $\Delta(p2) = \Delta(p1) = 1 = S(p2)$  (see figure 10, the transition  $a4 \rightarrow a5$  within the band 1).

That completes the proof of the claim and of the theorem.  $\square$

## Appendix B. Proof of theorem 3

*Proof.* If  $x > l$  there exists a topology under which a certain cell, say  $c$ , spans on at least 3 bands. Without loss of generality, let us denote the CIC-D indices of these three bands by 0, 1, 2 from west to east, respectively. Assuming that the convention used is that each cell transmits the code of the easternmost band on which it spans, we get that  $c$  transmits the code 2. Let  $d$  be the western neighbor of  $c$ , such that the easternmost band on which  $d$  spans, is the westernmost band on which  $c$  spans. That implies that  $d$  transmits the code 0. Upon movement from  $c$  to  $d$  the code changes from 2 to 0. This code transition is interpreted also as a movement of one band eastward, in contradiction to the movement from  $c$  to  $d$ , which is westward. Clearly, this contradiction still holds if the transmission convention is changed, such that each cell transmits the code of the westernmost band on which it spans. Hence, tracing the CIC-D indices cannot properly determine the user movement, and hence its location.  $\square$

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