Depth First Search

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- Assume unweighted directed graph (allow self loops)

DFS(G)

1. For each vertex $u \in G.V$
2. u.color = WHITE
3. u.π = NIL
4. time = 0
5. For each vertex $u \in G.V$
6. if u.color == WHITE
7. **DFS-Visit(G,u)**

DFS-Visit(G,u)

1. time = time + 1
2. **u.d = time**
3. **u.color = GRAY**
4. For each vertex $v \in G.Adj[u]$
5. if v.color == WHITE
6. v.π = u
7. **DFS-Visit(G,v)**
8. **u.color = BLACK**
9. time = time + 1
10. **u.f = time**

Each vertex has two timestamps:

$\boldsymbol{u.d}$: First discovered and GRAYed

$\boldsymbol{u.f}$: Finished and Blackened

$\boldsymbol{\forall u: 1 \leq u.d < u.f \leq 2n}$

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There is no special source s

We visit all vertices

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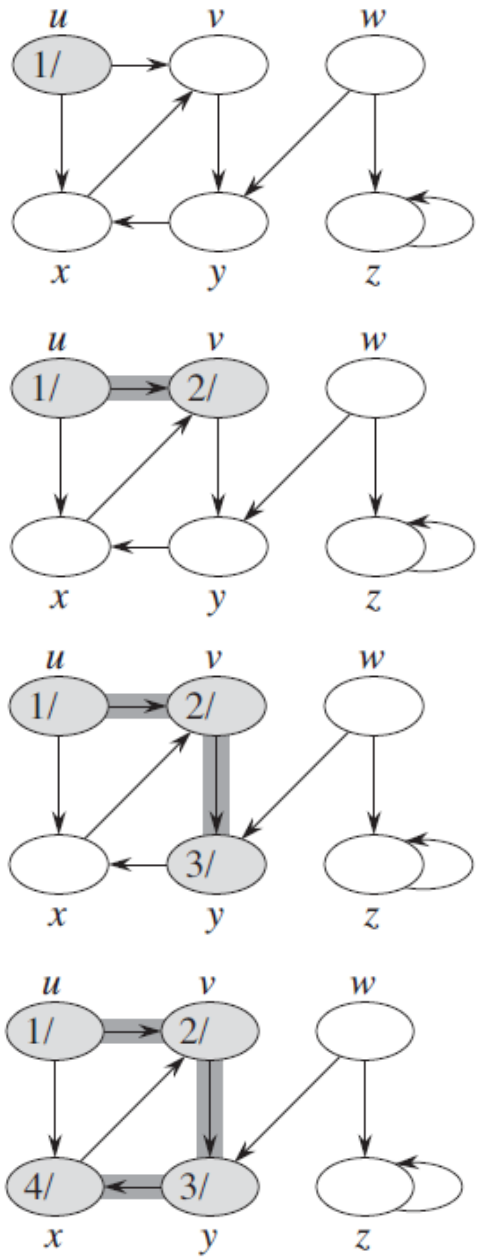

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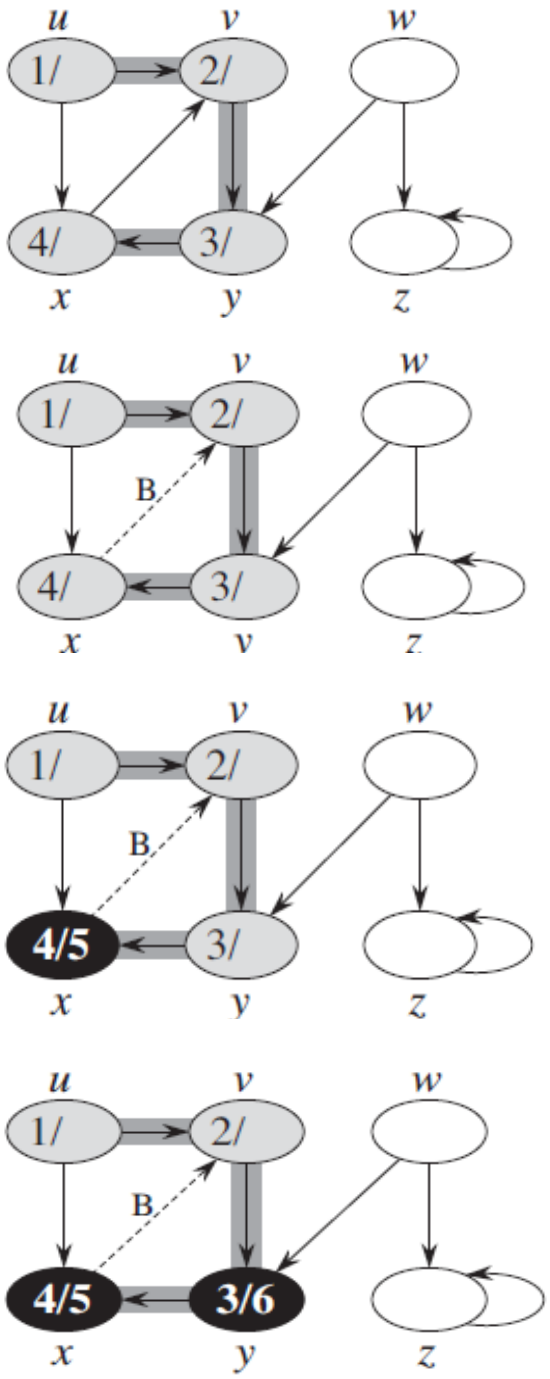

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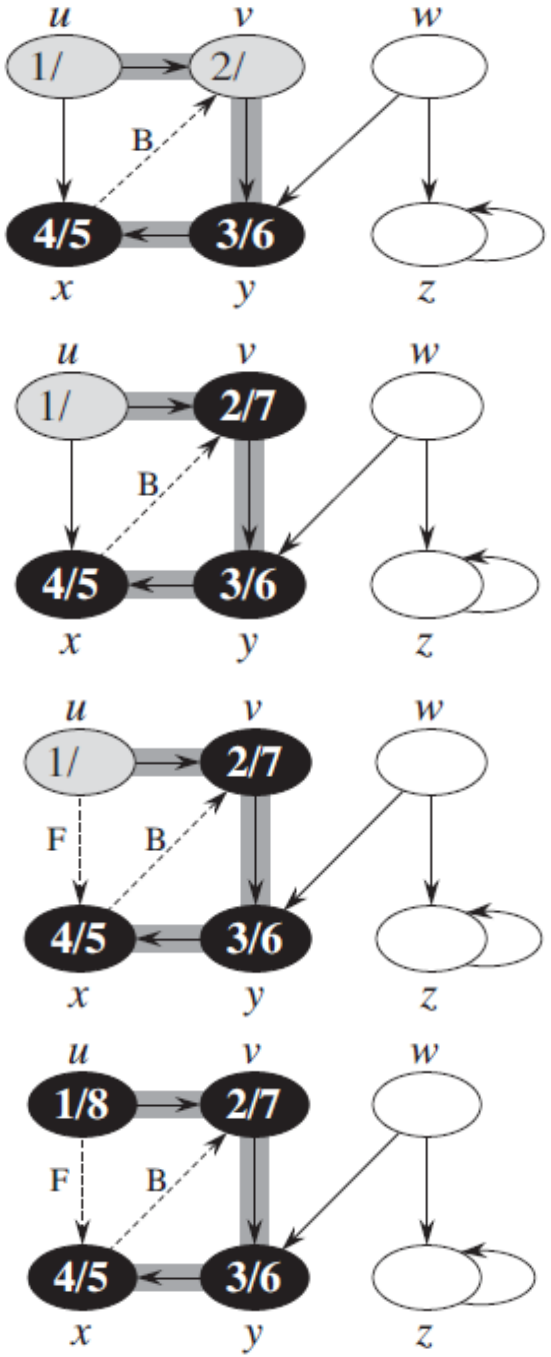

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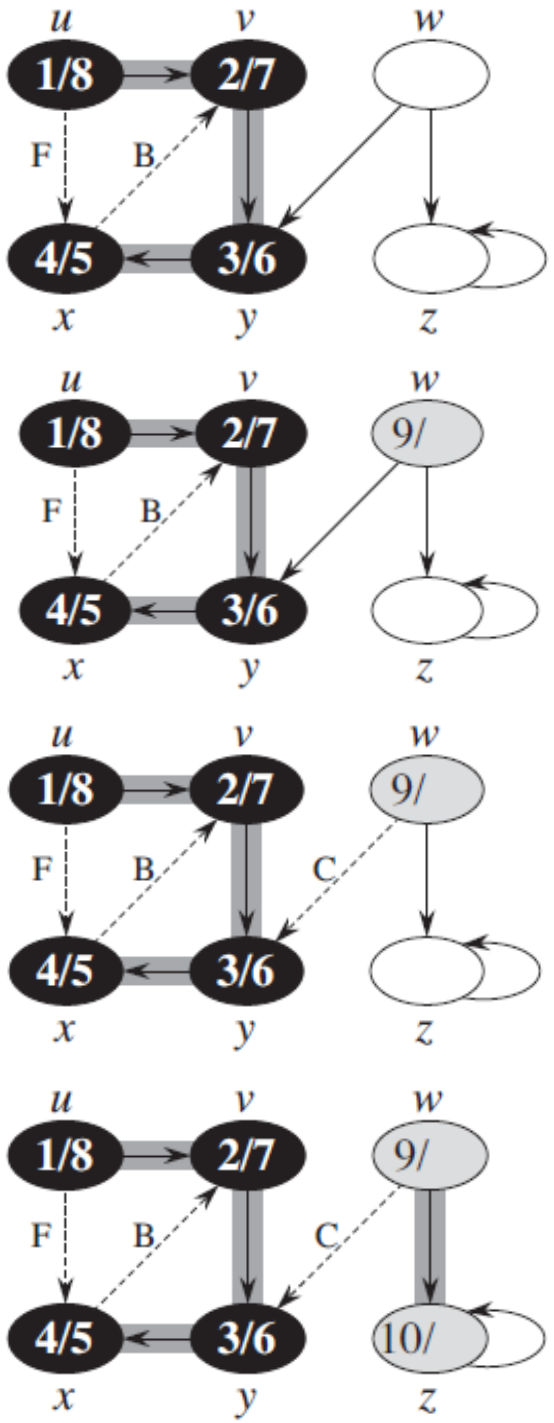

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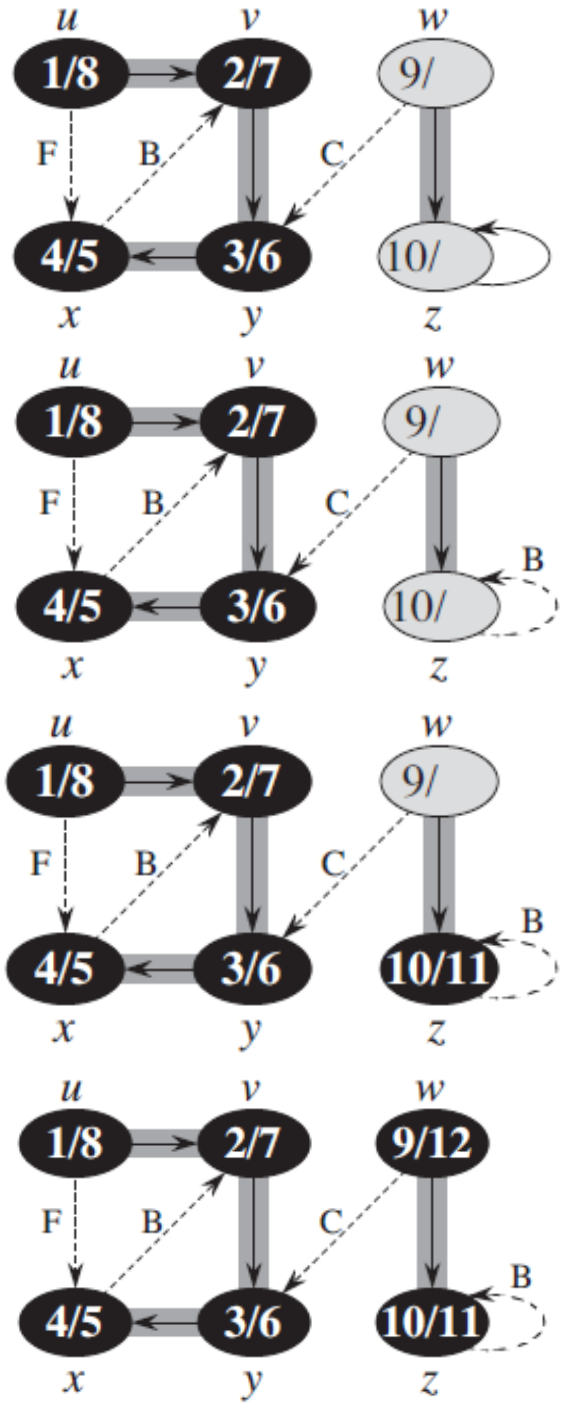

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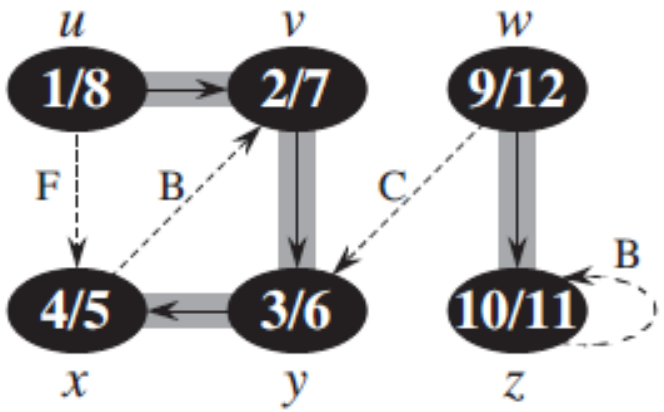
Lem 1: The graph $G_\pi = (V, E_\pi)$, $E_\pi = \{ (v_\pi, v) \mid v_\pi \neq NIL \}$ is a forest (DFS-forest)

Proof: It corresponds to the structure of the recursive calls to DFS-Visit □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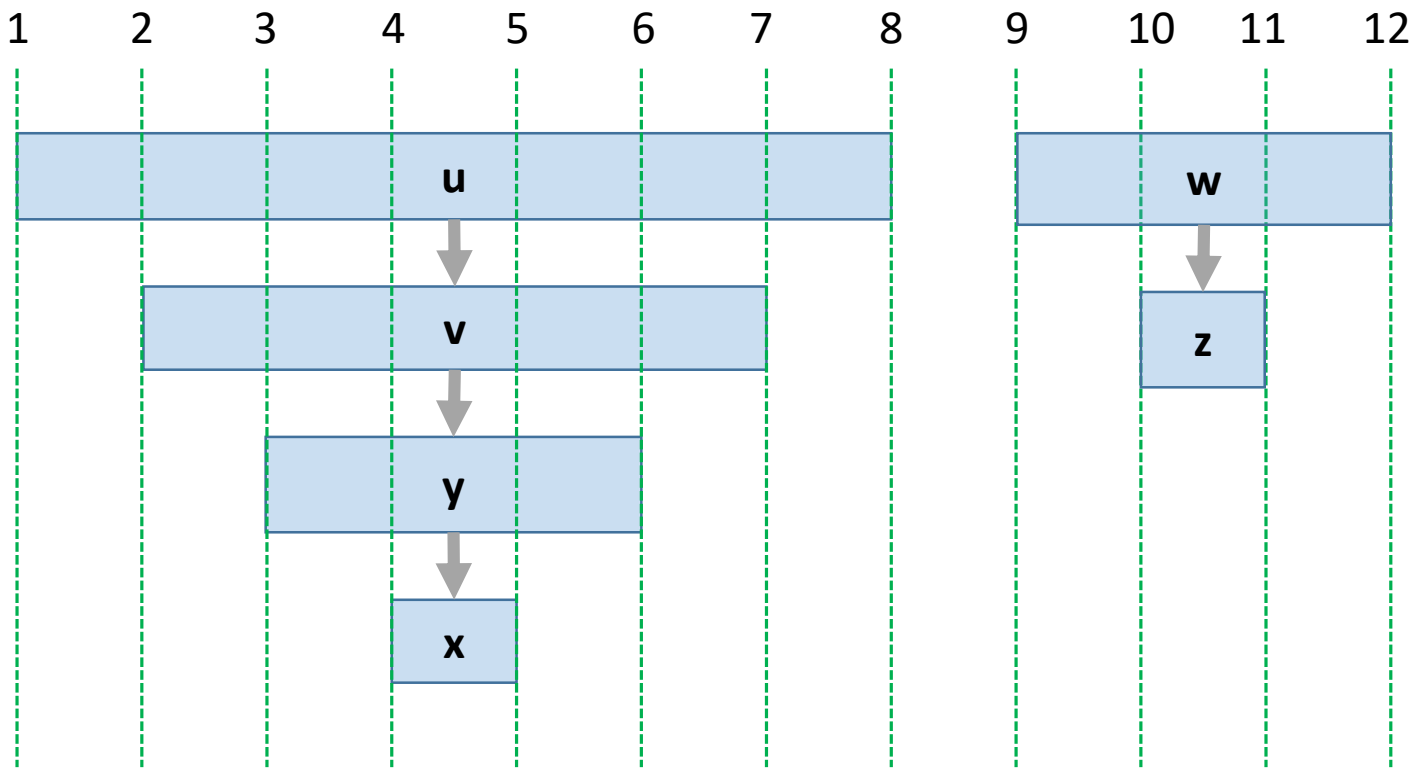
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Thm 2: $[v.d, v.f] \subset [u.d, u.f]$ iff v is a descendant of u in the DFS-forest, $[v.d, v.f] \cap [u.d, u.f] = \emptyset$ iff u and v are unrelated in the DFS-forest

Proof: Since the interval of v is the time in which the recursive call DFS-Visit(G,v) is active.

□

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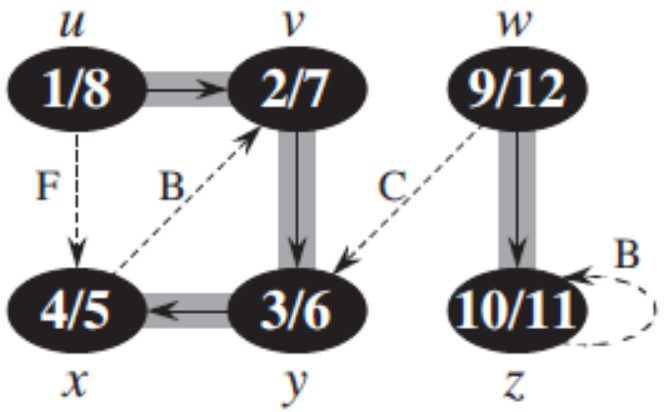
Thm 3: v is a descendant of u in the DFS-forest iff when we invoke DFS-Visit(u) there is a white path from u to v

Edge classification

- Tree edges: (u,v), DFS-Visit(v) was called from DFS-visit(u)
- Back edges: (u,v) such that v is an ancestor of u in the DFS-forest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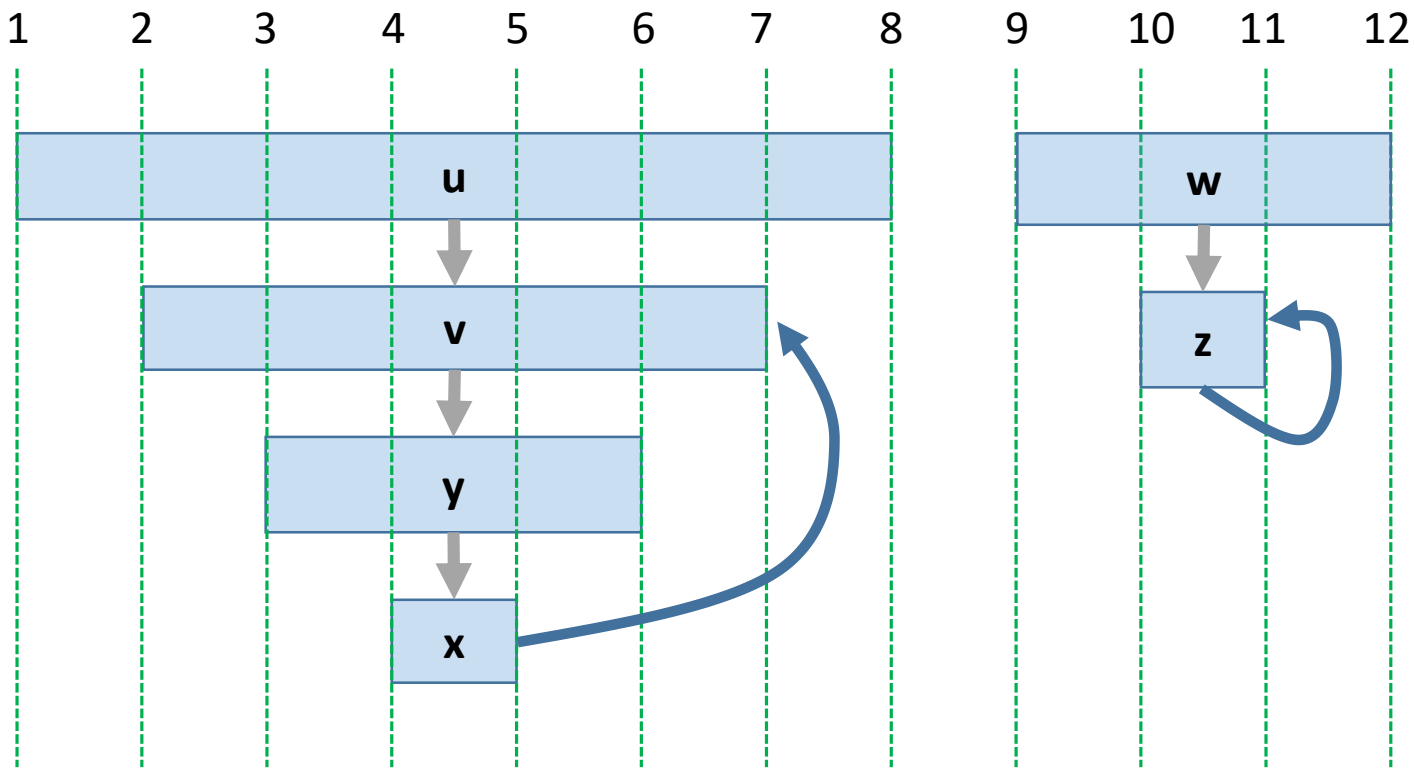
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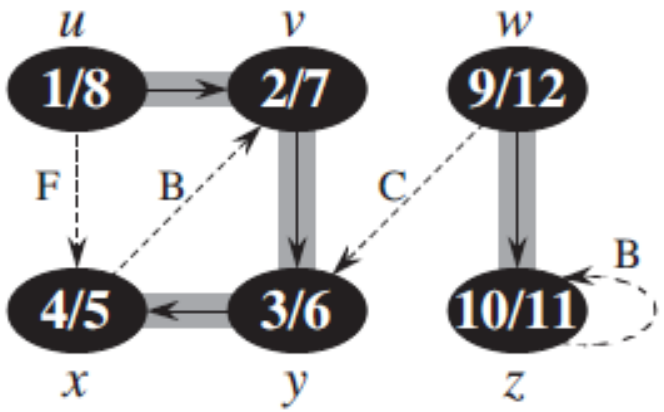
- Forward edges: nontree edges (u,v) such that v is a descendant of u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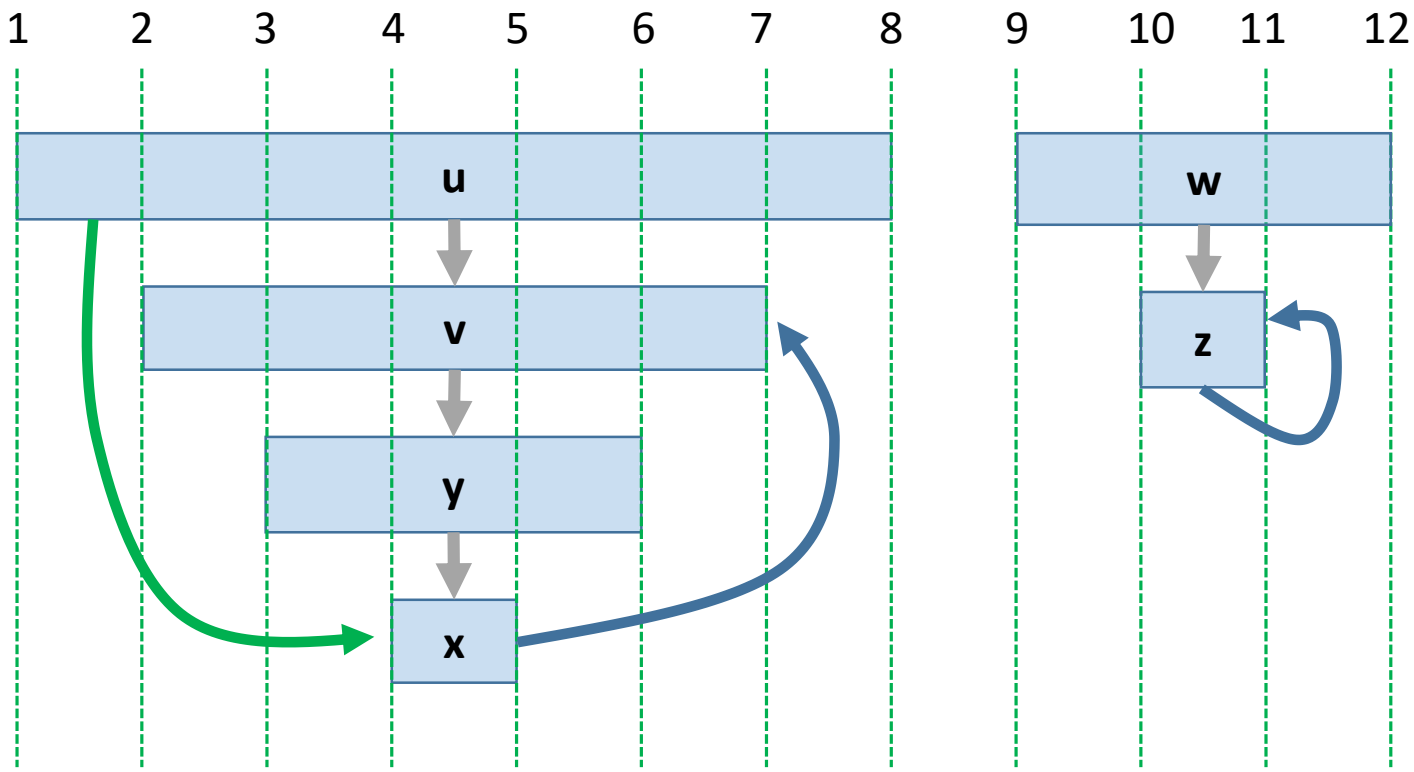
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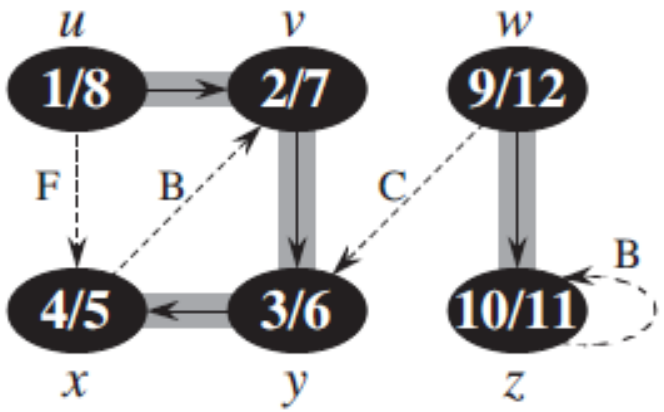
- Cross edges: all other edges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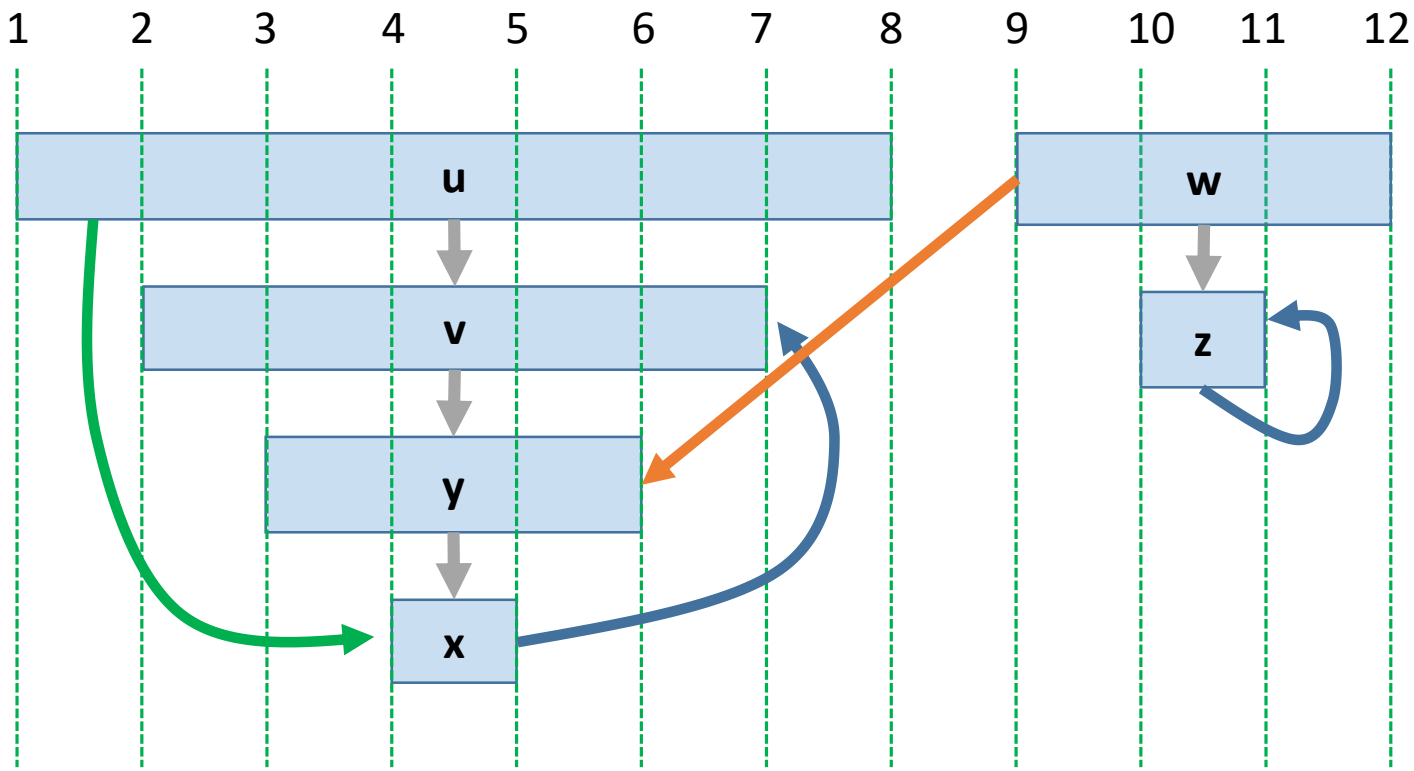
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Cross edges

Thm 4: If (u,v) is a cross edge then $u.d > v.f$

Classify as we go..

- Tree edges: (u,v), v is white
- Back edges: (u,v) v is gray $v.d < u.d$
- Forward edges: v is black $v.d > u.d$
- Cross edges: v is black, $v.d < v.f < u.d$