Time-Inconsistent Planning: A Computational Problem in Behavioral Economics

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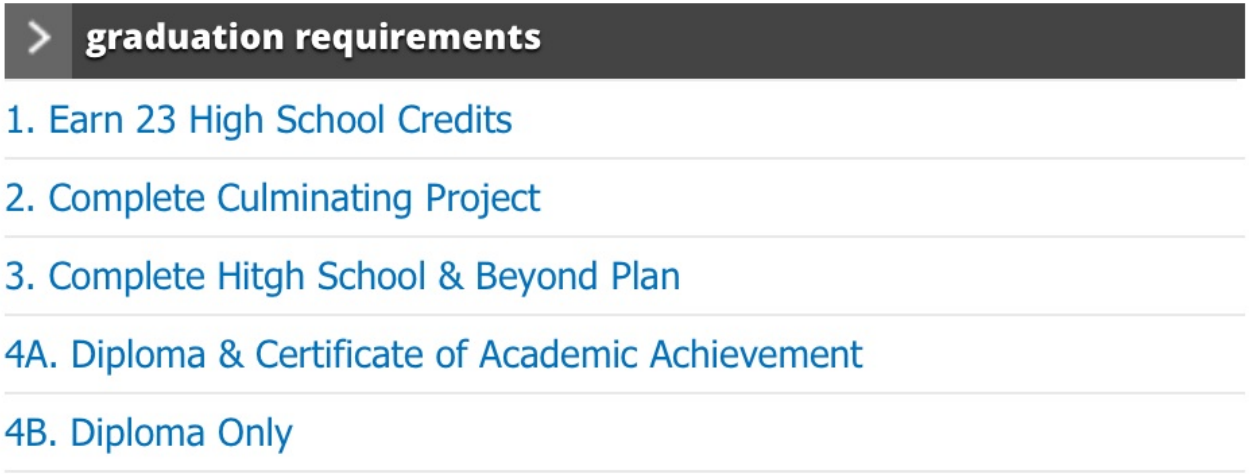
Cornell Hebrew Univ and MSR

arXiv:1405.1254

Planning and Time-Inconsistency



Tacoma Public School System

Fundamental behavioral process: Making plans for the future.

- Plans can be multi-step.
- Natural model: agents chooses optimal sequence given costs and benefits.

What could go wrong?

- Costs and benefits are unknown, and/or genuinely changing over time.
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Why did George Akerlof not make it to the post office?

Agent must ship a package sometime in next n days.

- One-time effort cost c to ship it.
- Loss-of-use cost x each day hasn't been shipped.

An optimization problem:

- If shipped on day t, cost is $c + tx$.
- Goal: $\min_{1 \le t \le n} c + tx$.
- Optimized at $t = 1$.

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In Akerlof's story, he was the agent, and he *procrastinated*:

- Each day he planned that he'd do it tomorrow.
- Effect: waiting until day n, when it must be shipped, and doing it then, at a significantly higher cumulative cost.

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A model based on present bias [Akerlof 91; cf. Strotz 55, Pollak 68]

- Costs incurred today are more salient: raised by factor $b > 1$.

On day t:

- Remaining cost if sent today is bc.
- Remaining cost if sent tomorrow is $bx + c$.
- Tomorrow is preferable if $(b-1)c > bx$.

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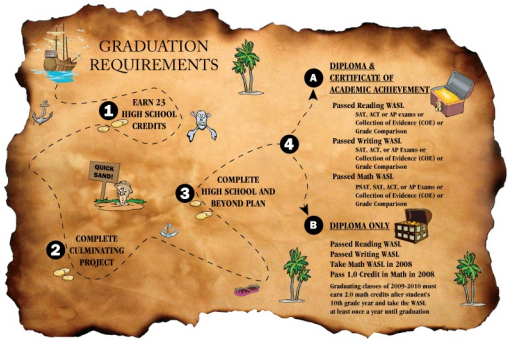
General framework: quasi-hyperbolic discounting [Laibson 1997]

- Cost/reward c realized t units in future has present value $\beta\delta^t c$
- Special case: $\delta = 1$, $b = \beta^{-1}$, and agent is naive about bias.
- Can model procrastination, task abandonment [O'Donoghue-Rabin08], and benefits of choice reduction [Ariely and Wertenbroch 02, Kaur-Kremer-Mullainathan 10]

Cost Ratio



Cost ratio:

$$\frac{\text{Cost incurred by present-biased agent}}{\text{Minimum cost achievable}}$$

Across all stories in which present bias has an effect, what's the worst cost ratio?

$$\max_{\text{stories } S} \text{cost ratio}(S).$$

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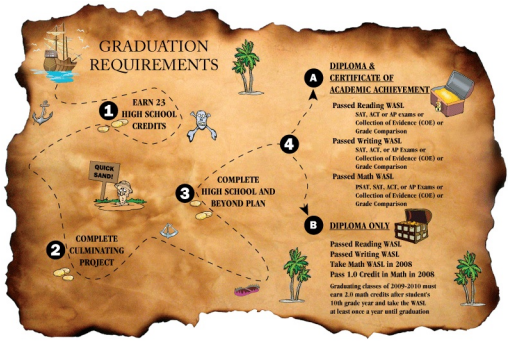

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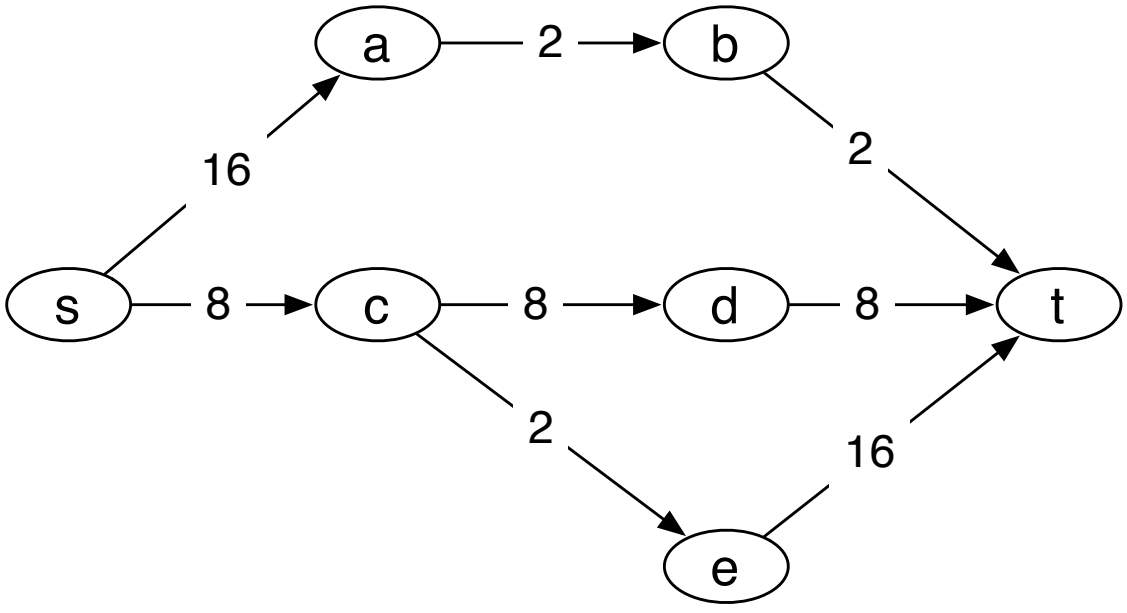
A Graph-Theoretic Framework



Use graphs as basic structure to represent scenarios.

- Agent plans to follow cheapest path from s to t.
- From a given node, immediately outgoing edges have costs multplied by $b > 1$.

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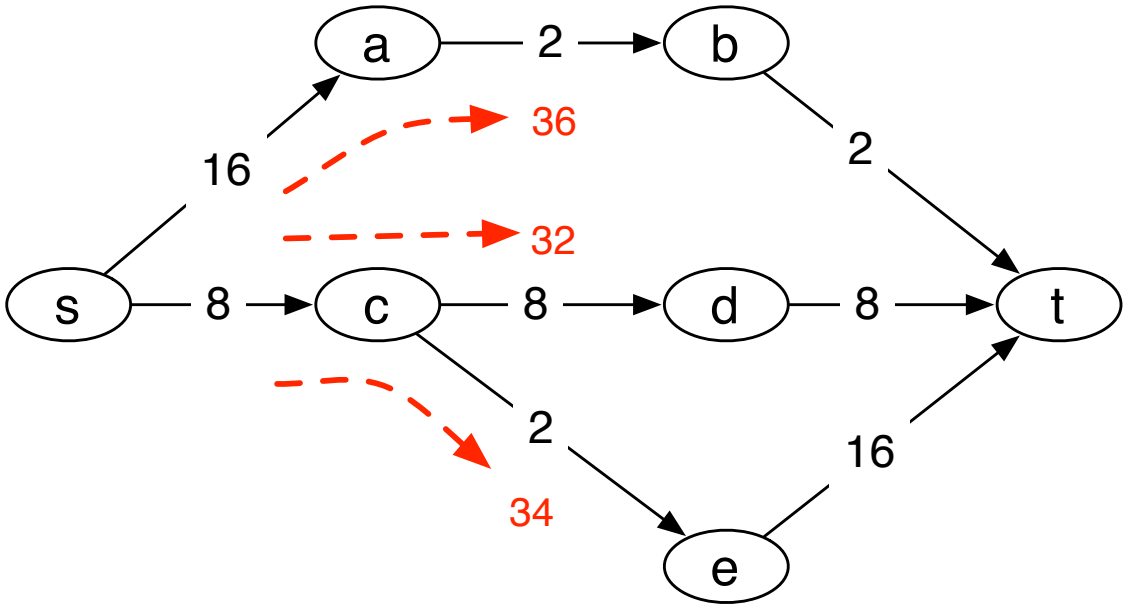

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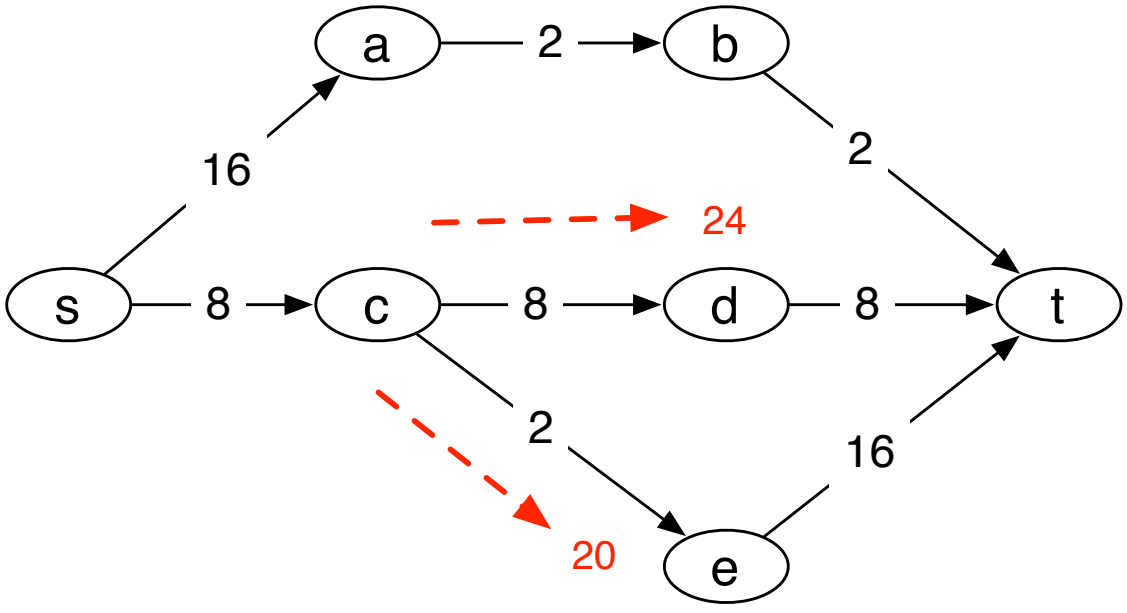

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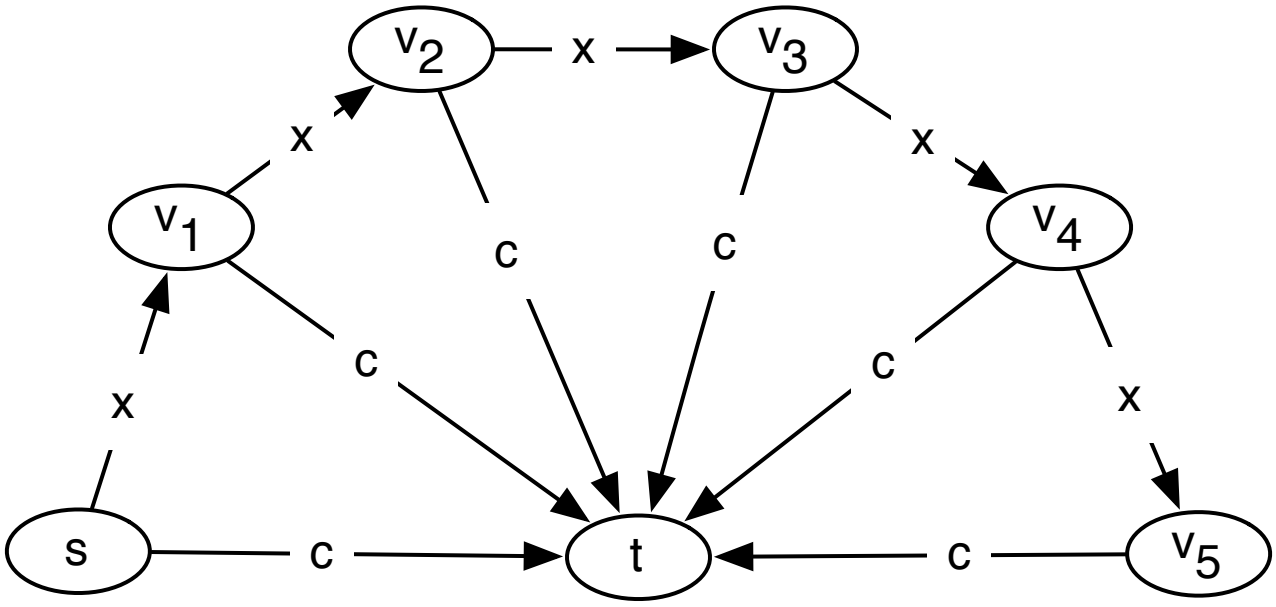
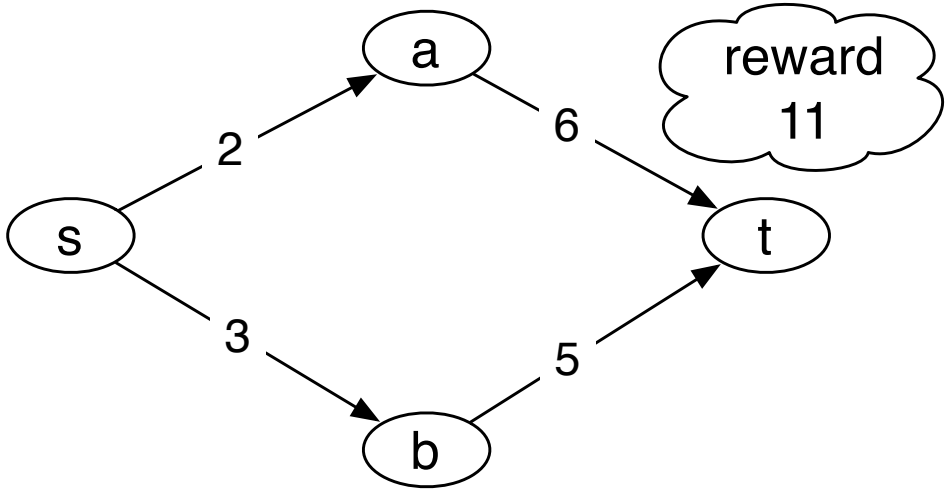
Example: Akerlof's Story as a Graph



Node v_i = reaching day i without sending the package.

Paths with Rewards



Variation: agent only continues on path if cost $\leq$ reward at t.

- Can model abandonment: agent stops partway through a completed path.
- Can model benefits of choice reduction: deleting nodes can sometimes make graph become traversable.

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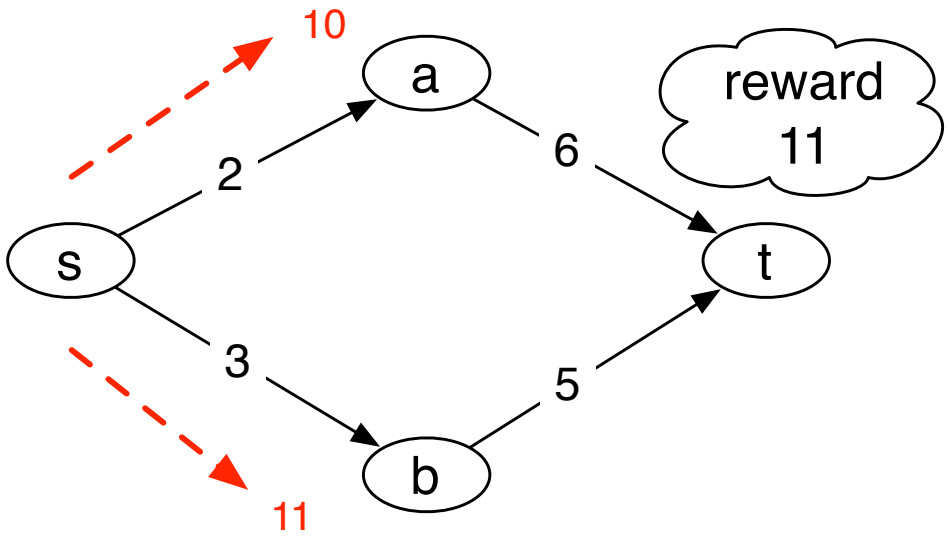

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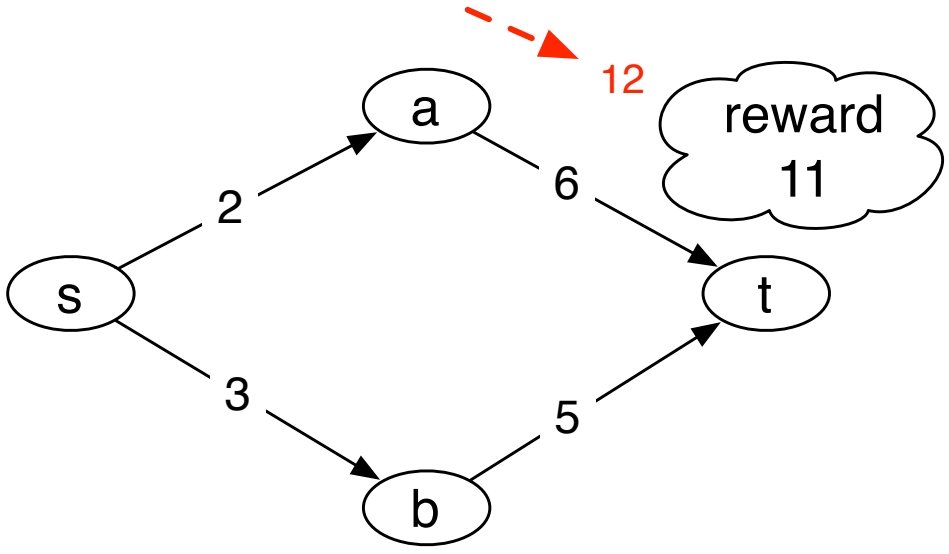

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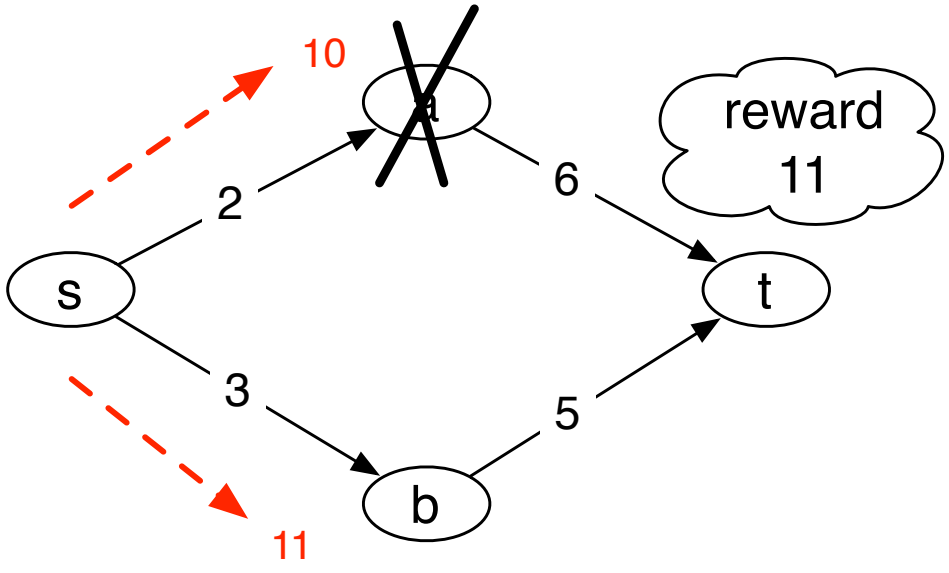

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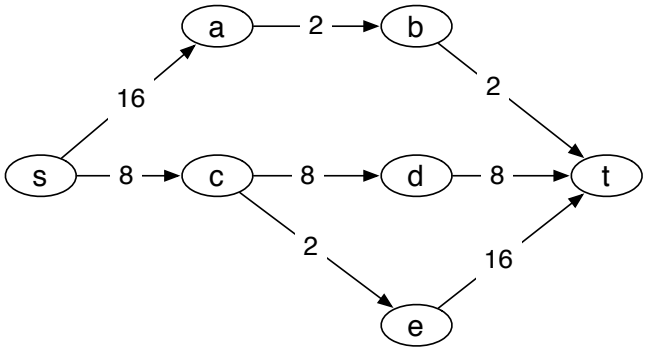
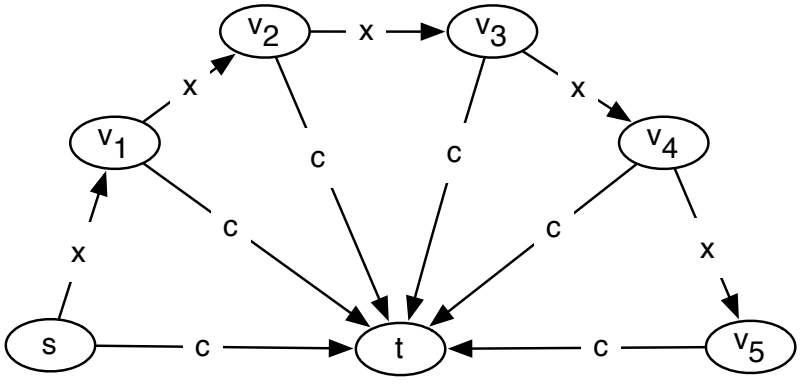
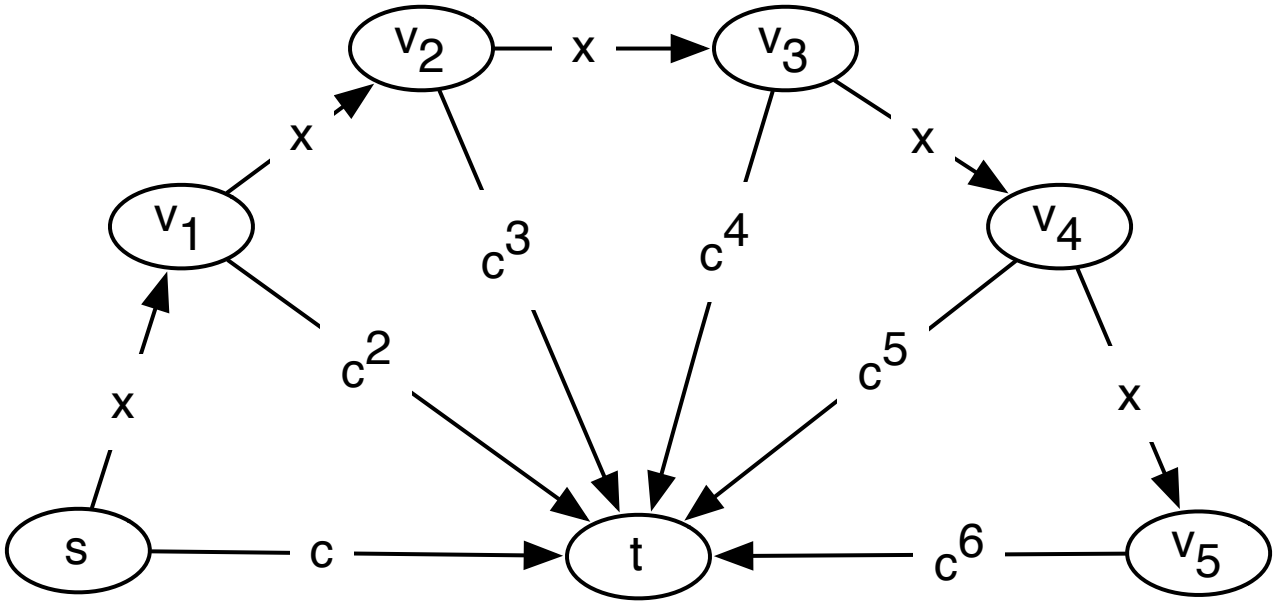
Overview





1. Analyzing present-biased behavior via shortest-path problems.
2. Characterizing instances with high cost ratios.
3. Algorithmic problem: optimal choice reduction to help present-biased agents complete tasks.
4. Heterogeneity: populations with diverse values of b.

A Bad Example for the Cost Ratio

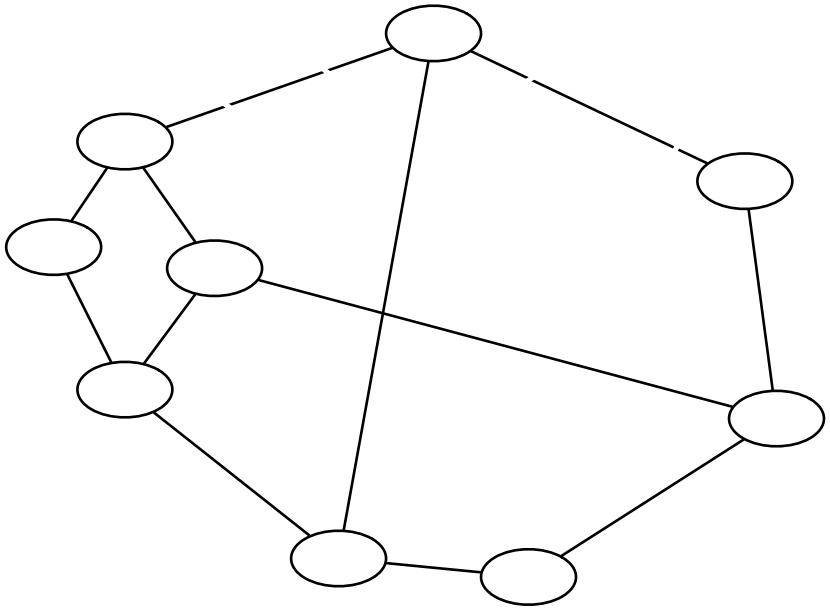


Cost ratio can be roughly b^n, and this is essentially tight.

Can we characterize the instances with exponential cost ratio?

- Goal, informally stated: Must any instance with large cost ratio contain Akerlof's story as a sub-structure?

Characterizing Bad Instances via Graph Minors



Graph H is a *minor* of graph G if
we can contract connected subsets of G into "super-nodes"
so as to produce a copy of H.

- In the example: G has a K_4-minor.

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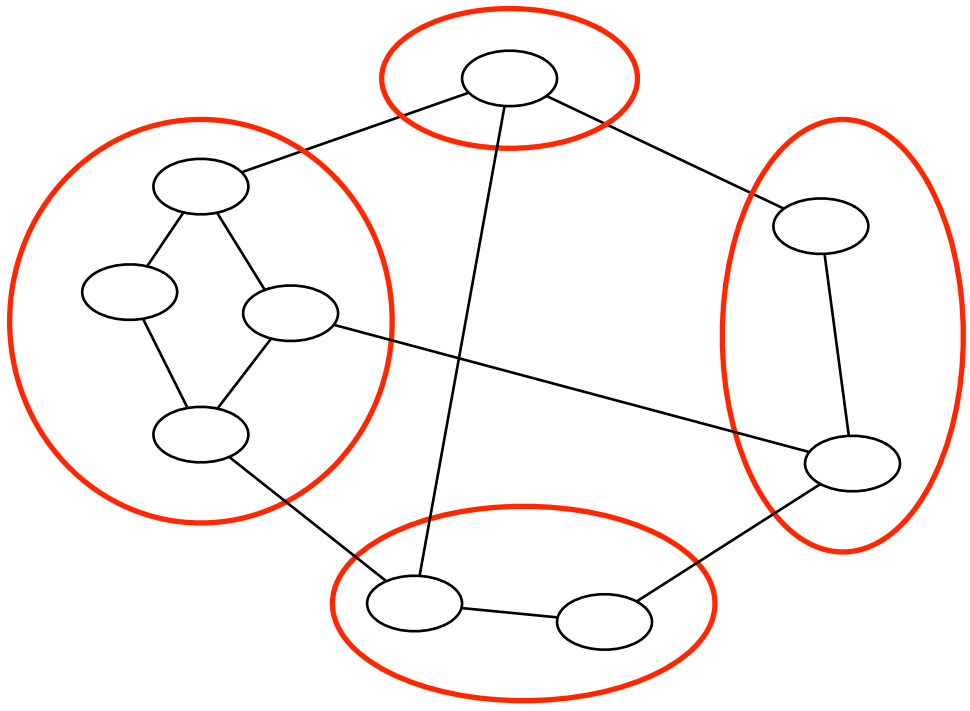

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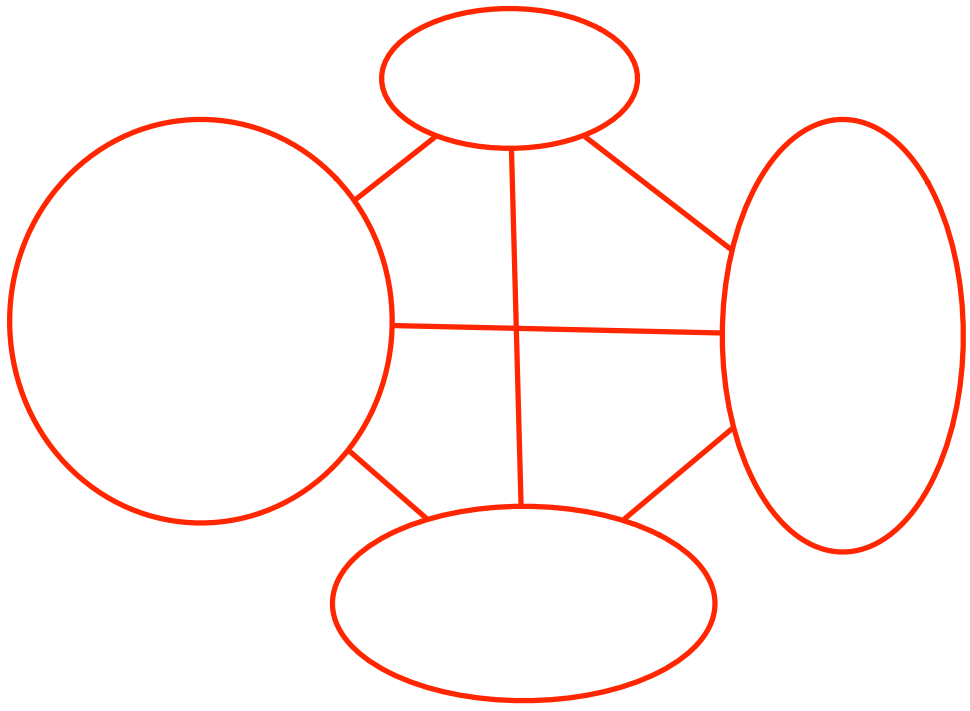

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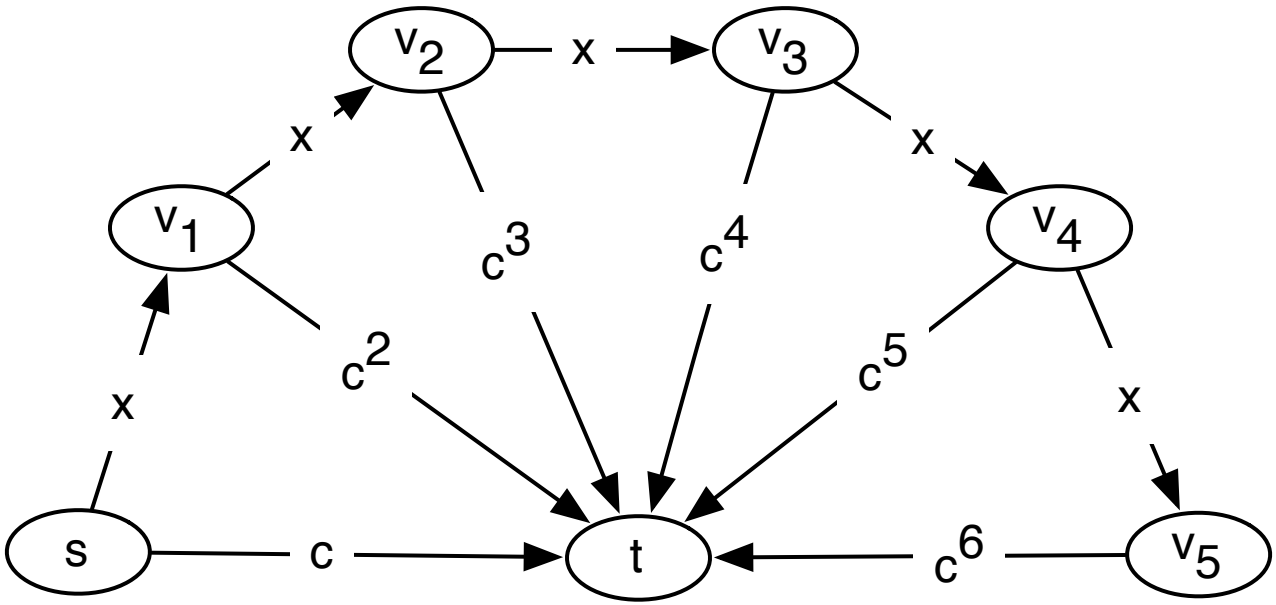

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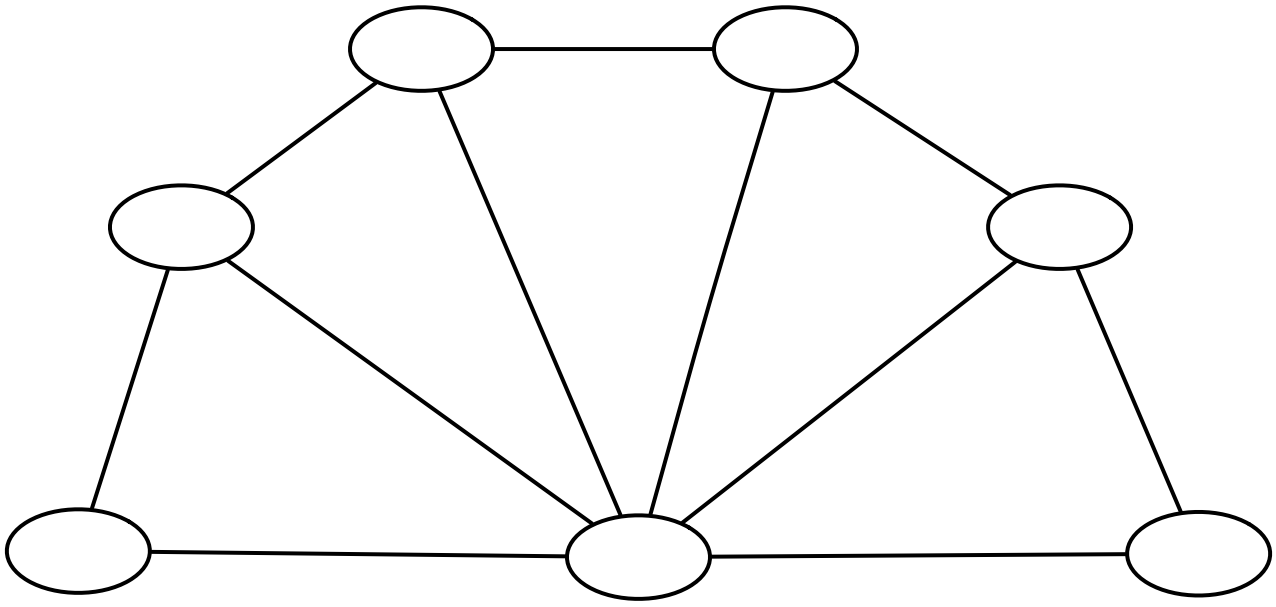
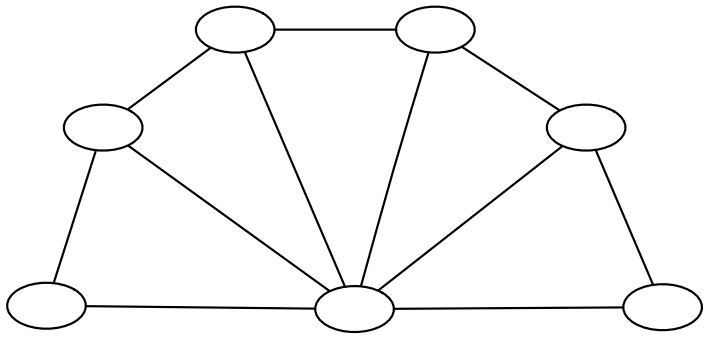


Characterizing Bad Instances via Graph Minors

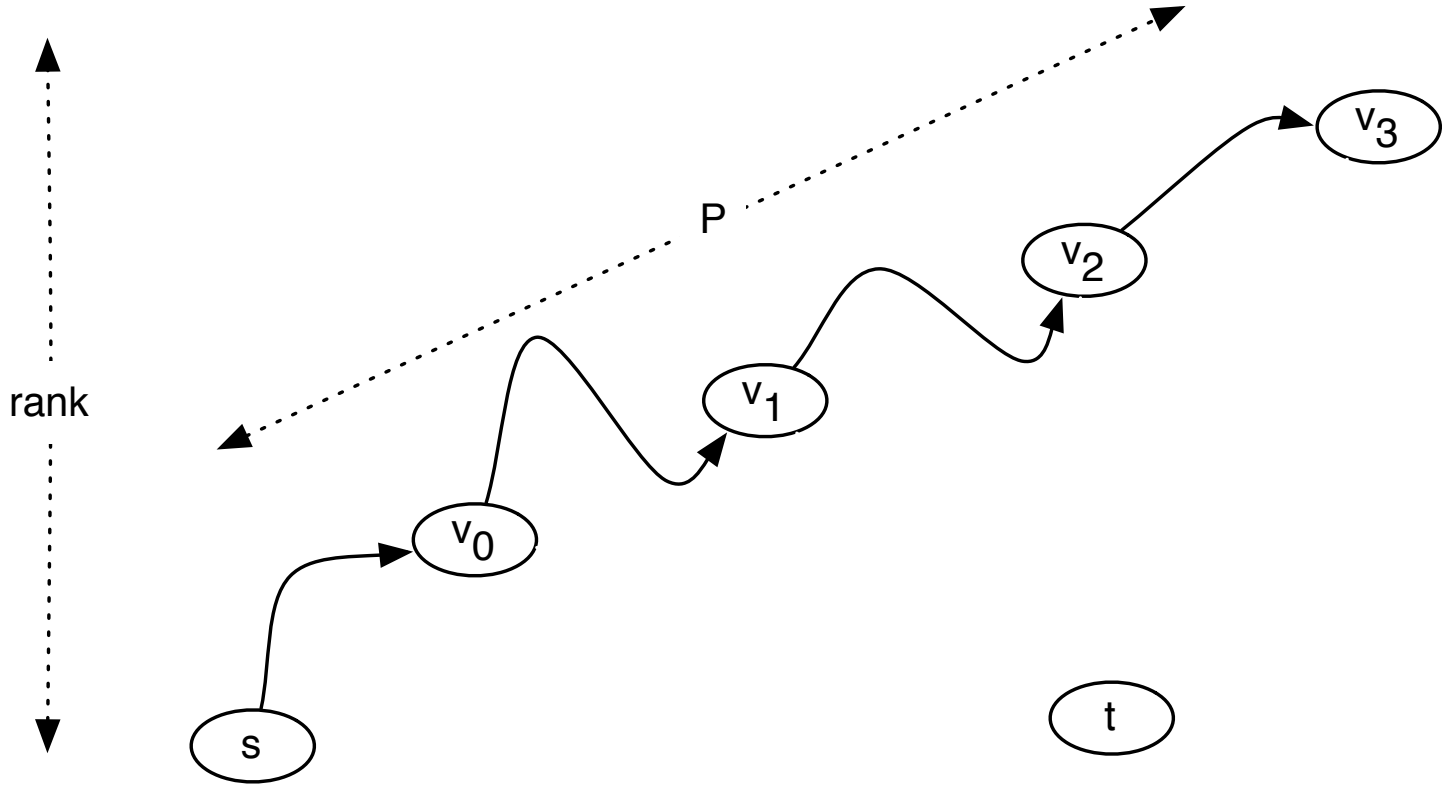
The k-fan $\mathcal{F}_k$: the graph consisting of a k-node path, and one more node that all others link to.



Theorem

For every $\lambda > 1$ there exists $\varepsilon > 0$ such that
if the cost ratio is $> \lambda^n$,
then the underlying undirected graph of the instance contains an $\mathcal{F}_k$-minor for $k = \varepsilon n$.

Sketch of the Proof



- The agent traverses a path P as it tries to reach t.
- Let the *rank* of a node on P be the logarithm of its dist. to t.
- Show that every time the rank increases by 1, we can construct a new path to t that avoids the traversed path P.

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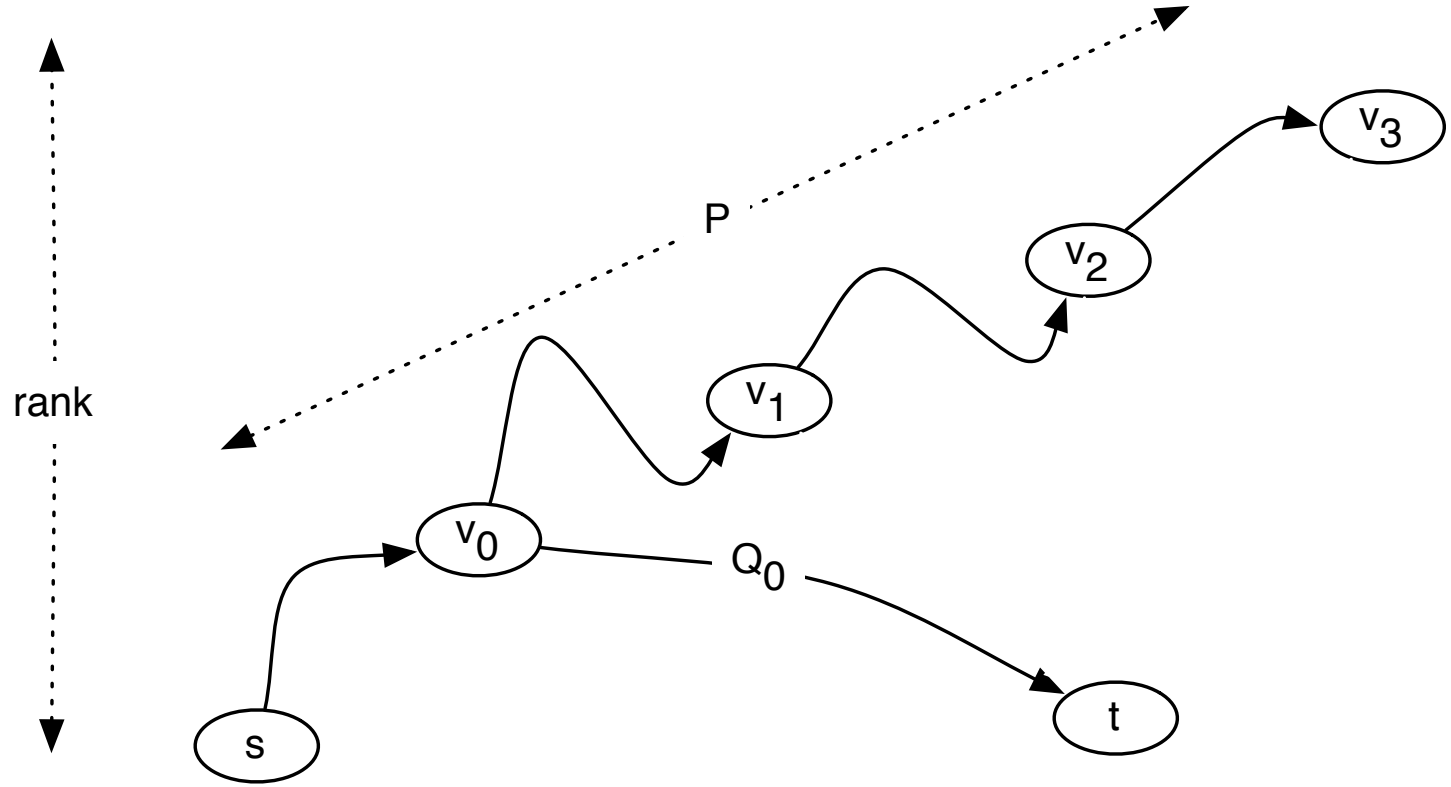

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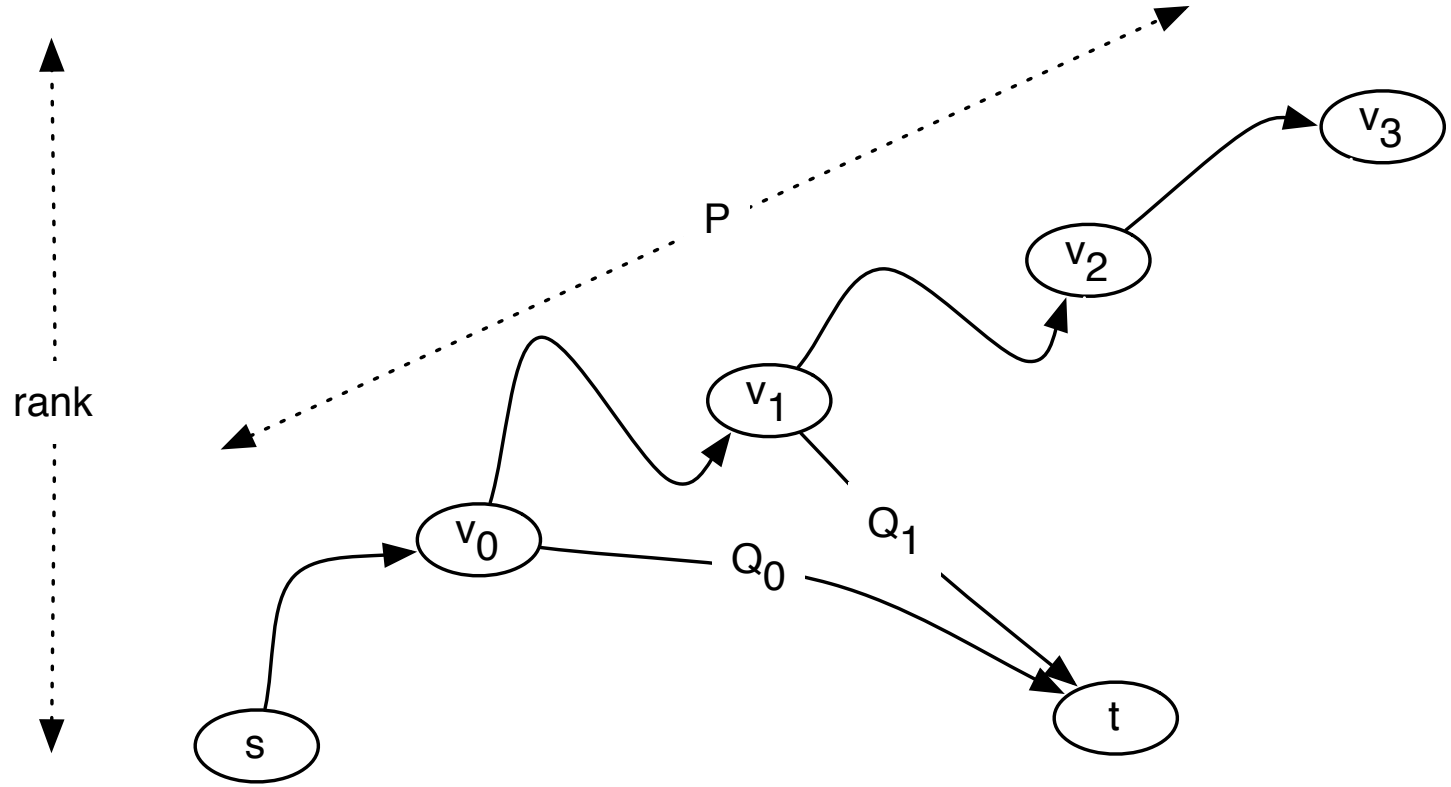

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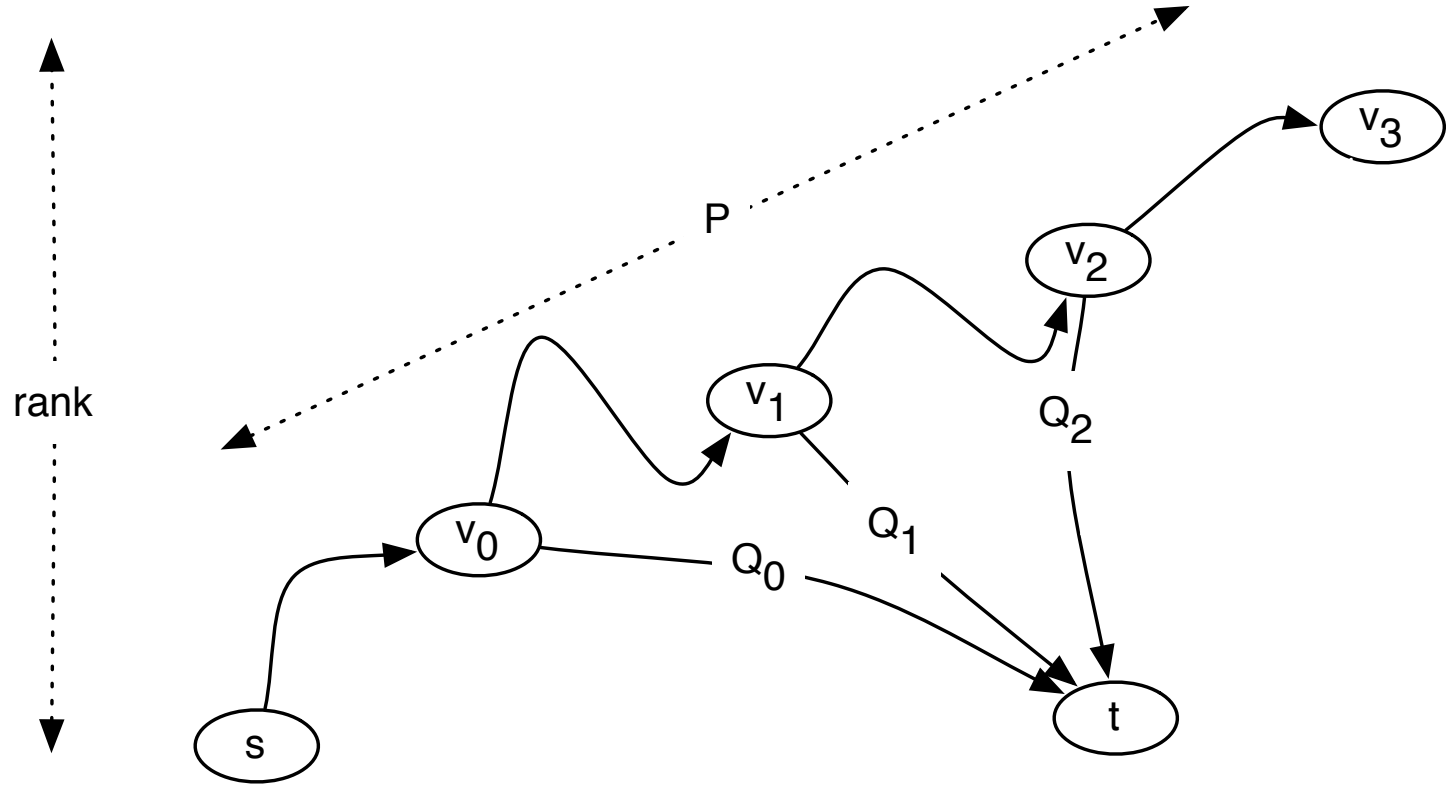

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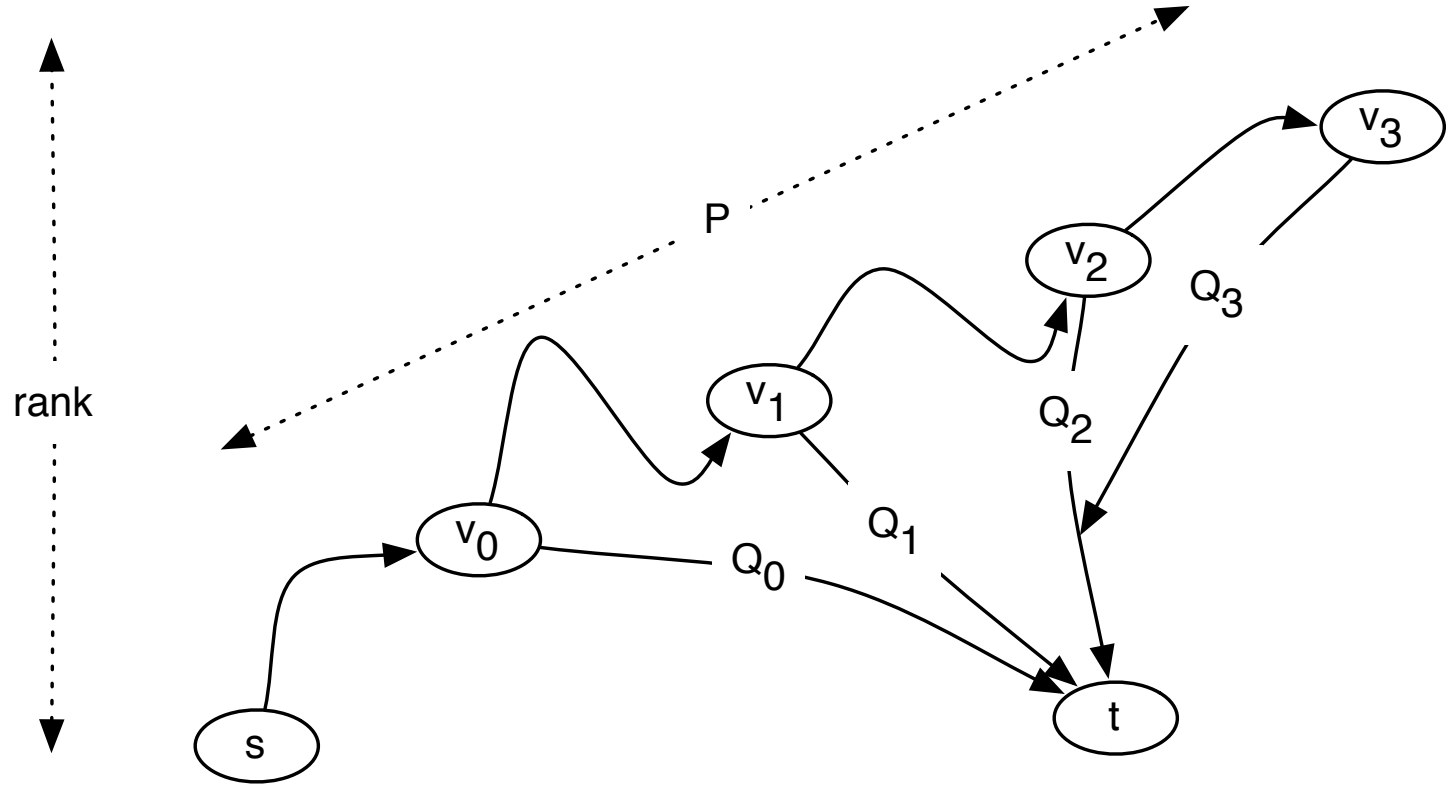

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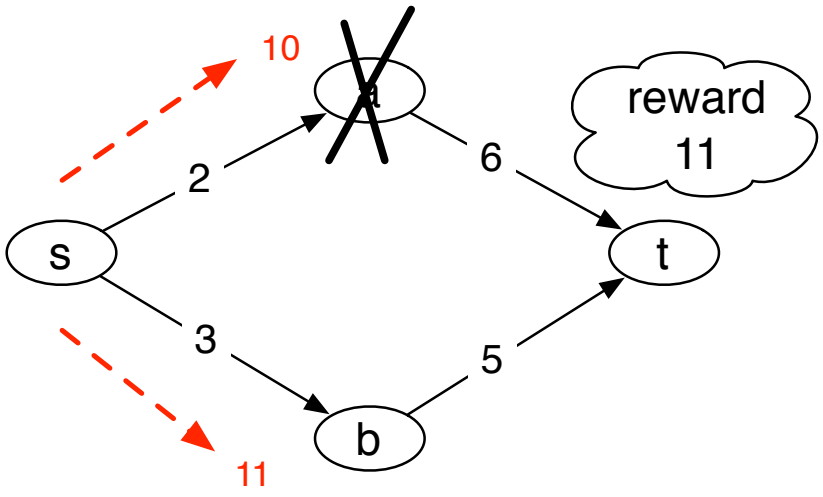
Choice Reduction



Choice reduction problem: Given G, not traversable by an agent, is there a subgraph of G that is traversable?

- Our initial idea: if there is a traversable subgraph in G, then there is a traversable subgraph that is a path.
- But this is not the case.

Results:

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- Open: can one find a traversable subgraph of G in polynomial time?

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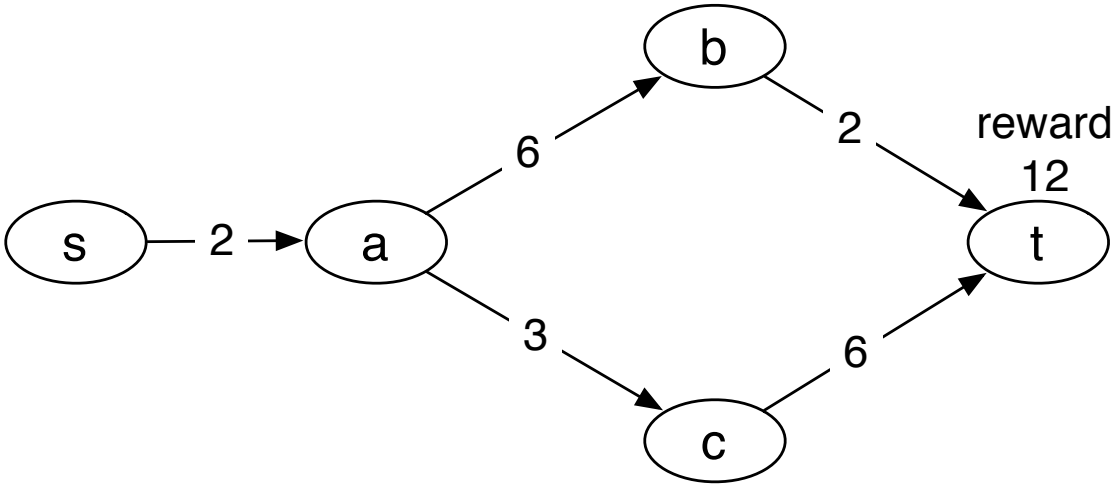

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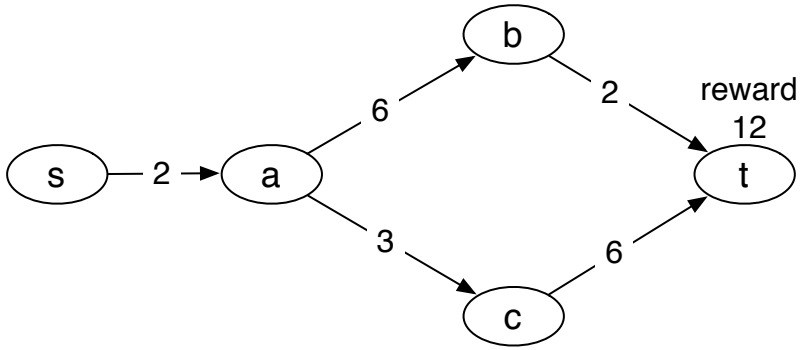
Further Directions



- Theorem on heterogeneity: In any instance, there are $O(n^2)$ combinatorially distinct choices of present-bias parameter b.
- Open: Finding a traversable subgraph in polynomial time?
- Open: A graph-minor characterization for small cost ratios? If the cost ratio is $> r$, is there an $\mathcal{F}_k$-minor for $k = f(r)$?
- Open: Polynomial-time algorithm to optimally place rewards at internal nodes of an instance?
- Connections to badge design? [Easley-Ghosh13, Anderson-Huttenlocher-Kleinberg-Leskovec13, Immorlica-Stoddard-Syrgkanis14]