PARALLEL COORDINATES : *VISUAL* Multidimensional Geometry and its Applications

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The Plane $\mathbb{R}^2$ with $\|$-coords

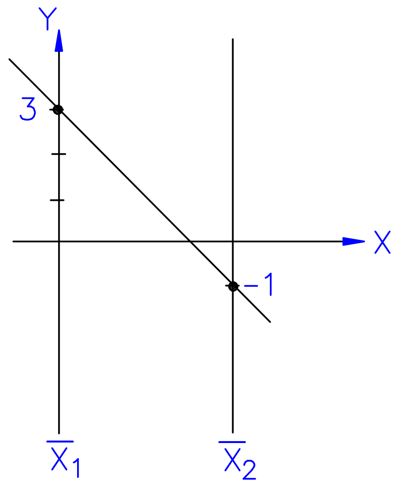

Figure 1: Points, above $(3, -1)$, on the plane are represented by lines.

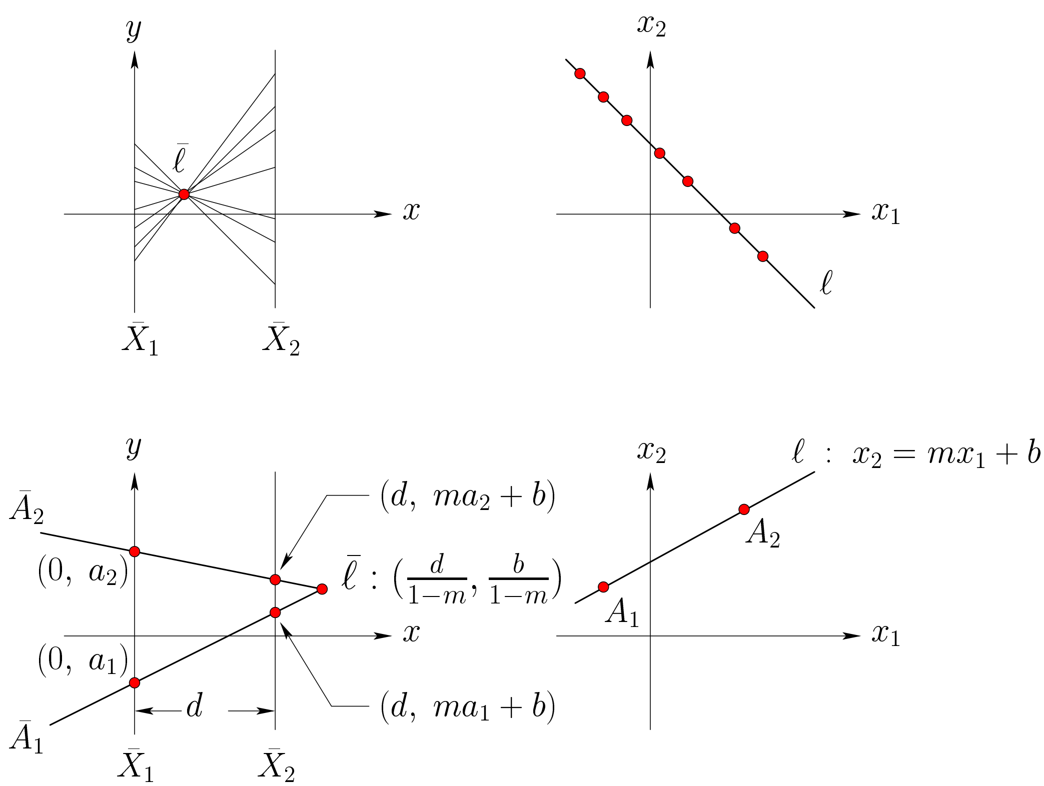

Figure 2: Conversely, lines are represented by points inducing a point $\longleftrightarrow$ line duality.

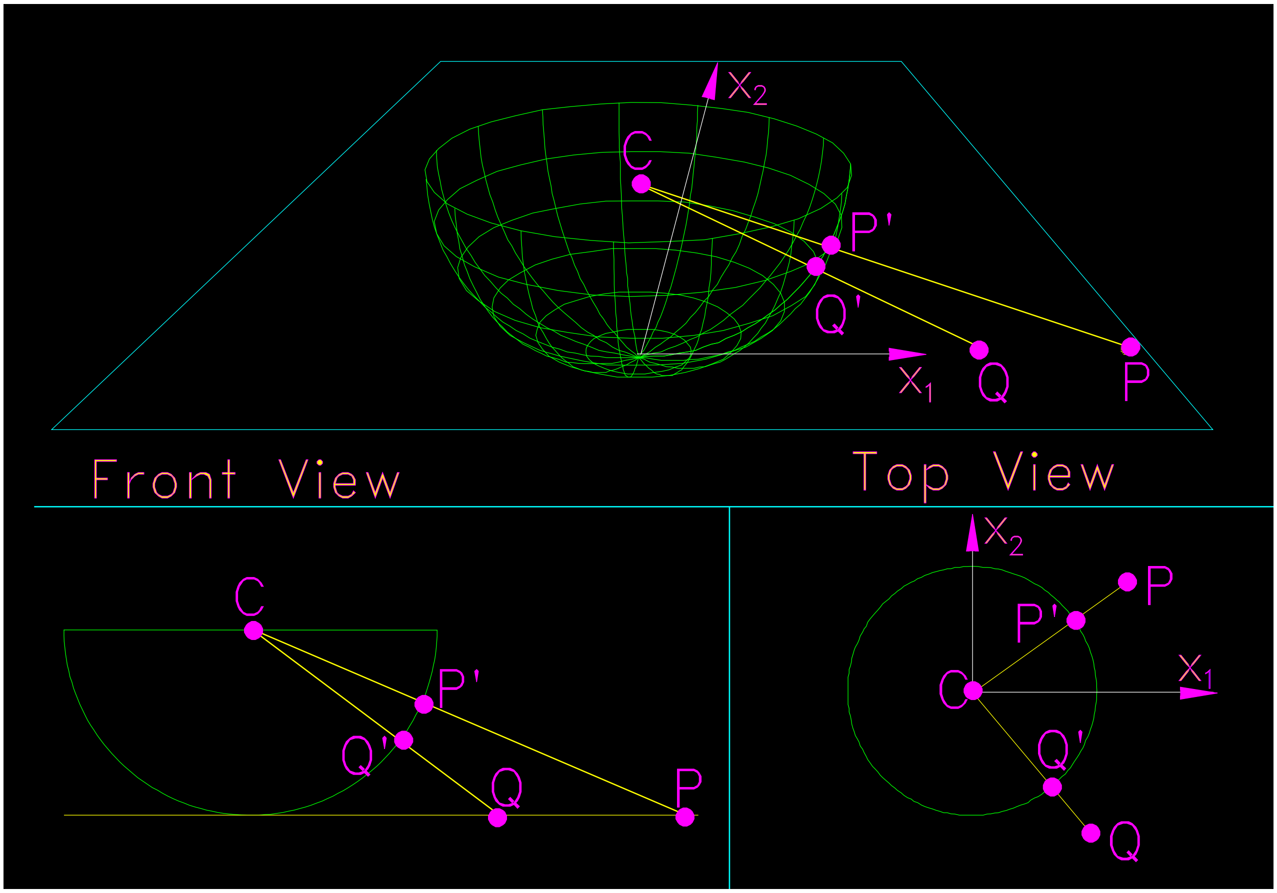

Figure 3: Model of the Projective Plane. Euclidean points are mapped into surface points of the hemisphere and *ideal points/directions* are mapped into the diameters of the "cap" with the same direction.

With d the distance between the axes the correspondence is :

$$\textit{line} \quad \ell : x_2 = m x_1 + b \quad \longleftrightarrow \quad \textit{point} \quad \bar{\ell} : \left(\frac{d}{1-m}, \frac{b}{1-m}\right) \qquad m \neq 1 \,. \tag{1}$$

Lines with negative slope $m < 0$ (negative correlation) are mapped into points between the axes, $m > 1$ to the left of the $\bar{X}_1$ and $0 < m < 1$ to the right of the $\bar{X}_2$ axes. To include lines with $m = 1$ the Euclidean plane $\mathbb{R}^2$ is embedded in the Projective plane $\mathbb{P}^2$. Then a line with slope $m = 1$ is mapped in the *direction* also called *ideal point* with slope b/d.

Homogeneous coordinates are very convenient and the conversion to/from Cartesian is easy i.e. *Cartesian* $\quad (a, b) \rightarrow (a, b, 1) \rightarrow k(a, b, 1) \quad \textit{for } k \neq 0$.

Sometimes it is preferable to describe the line ℓ by :

$$\ell : a_1 x_1 + a_2 x_2 + a_3 = 0 \tag{2}$$

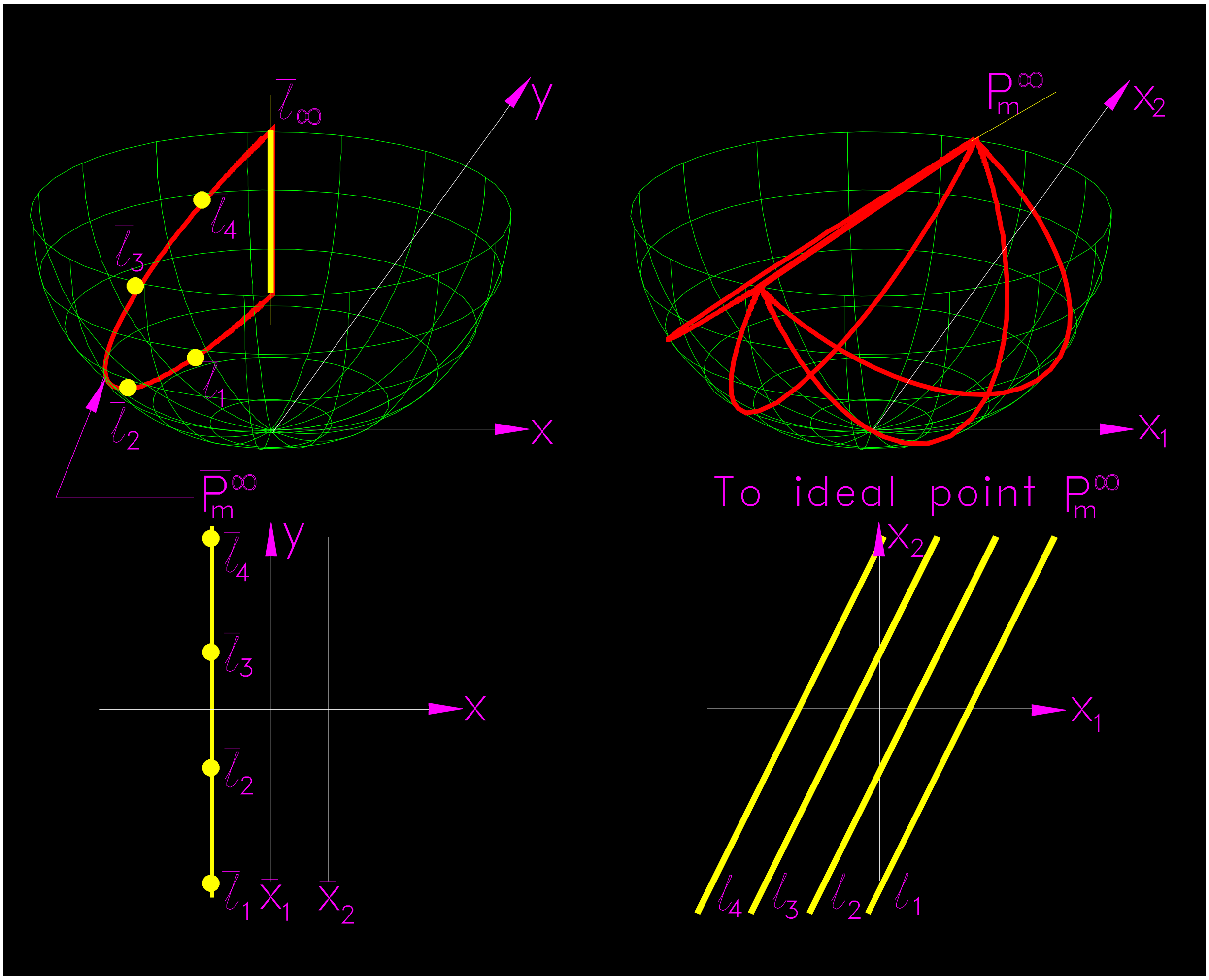

Figure 4: Under the duality parallel lines map into points on the same vertical line. On the projective plane model, the great semi-circles representing the lines share the same diameter since the lines have the same ideal point (direction). An ideal point in the direction with slope m is mapped into the vertical line $\bar{P}_m^\infty$.

and for $a_2 \neq 0$, $m = -\frac{a_1}{a_2}$ and $b = -\frac{a_3}{a_2}$, providing the correspondence :

$$\ell : [a_1, a_2, a_3] \longrightarrow \bar{\ell} : (da_2, -a_3, a_1 + a_2). \tag{3}$$

In turn this specifies a linear transformation between the triples ℓ and $\bar{\ell}$:

$$\bar{\ell} = Al \quad , l = A^{-1}\bar{\ell},$$

where ℓ and $\bar{\ell}$ are considered as column vectors. The 3×3 matrix is :

$$A \ = \begin{bmatrix} 0 & d & 0 \\ 0 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}, \ A^{-1} \ = \begin{bmatrix} -1/d & 0 & 1 \\ 1/d & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}. \tag{4}$$

which can be easily computed by taking 3 simple triples, like for example, [1,0,0], [0,1,0] and [0,0,1] for ℓ. For the other half of the duality, we look into the point $P \to \bar{P}$ line correspondence which is given by:

$$P : (p_1, p_2, p_3) \longrightarrow \bar{P} : [(p_1 - p_2), dp_3, -dp_1]. \tag{5}$$

Again taking P and $\bar{P}$ as column vectors we have:

$$\bar{P} = B^{-1}P \ , P = B\bar{P}$$

with

$$B^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & -d \\ d & 0 & 0 \end{bmatrix} , \ B = \begin{bmatrix} 0 & 0 & 1/d \\ 1 & 0 & 1/d \\ 0 & -1/d & 0 \end{bmatrix} . \tag{6}$$

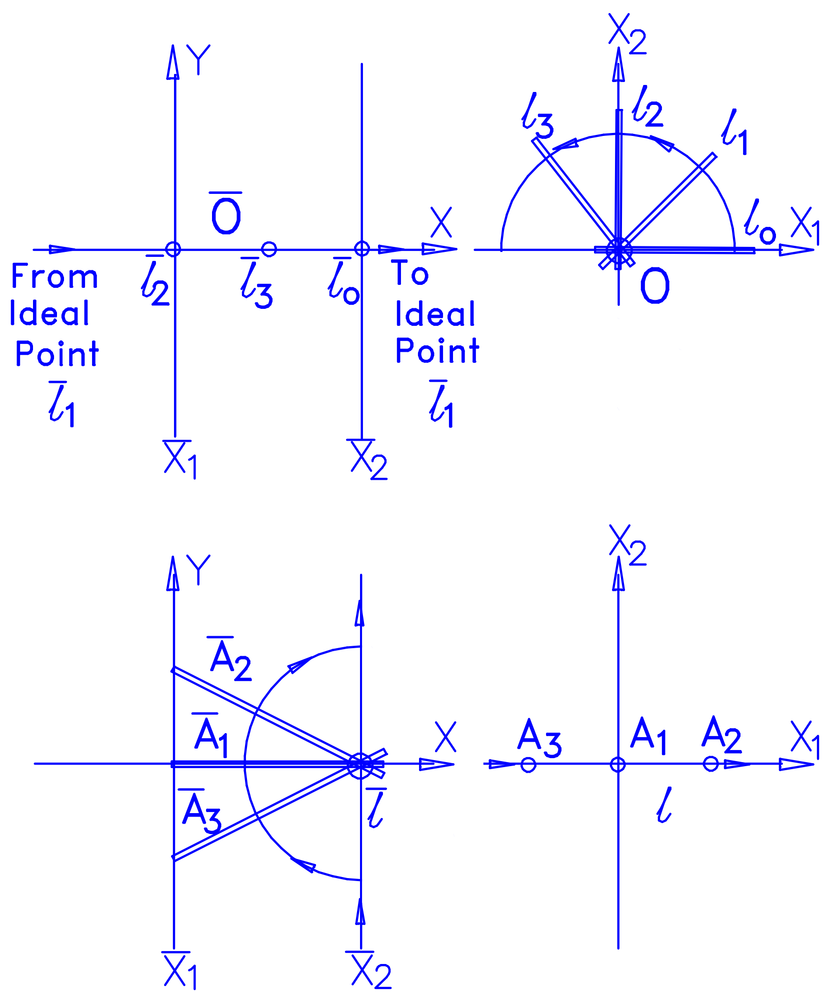

Figure 5: Duality : Rotation of a line about a point $\leftrightarrow$ Translation of a point on a line.

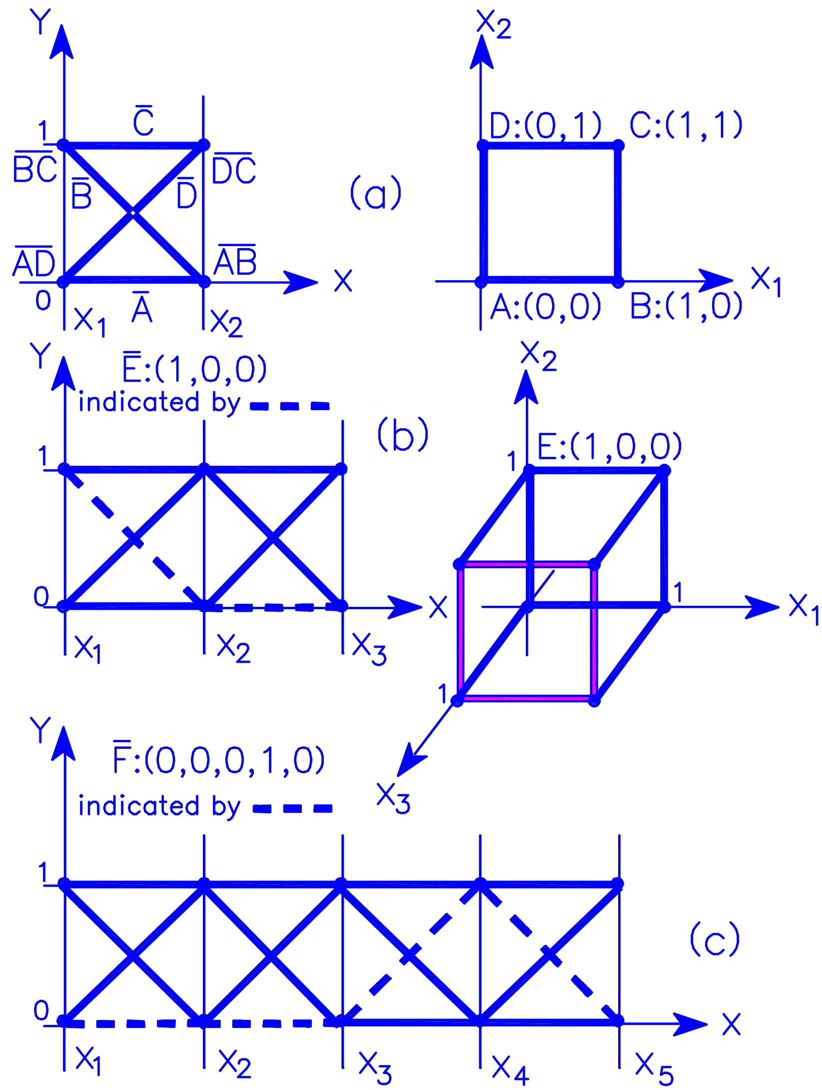

Figure 6: (a)Square,(b) 3-D cube (c) 5-D hypercube all with unit side. All vertices, edges, faces of all order can be seen – after learning the contents of sections Lines & Planes.

References

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