Introduction to Modern Cryptography

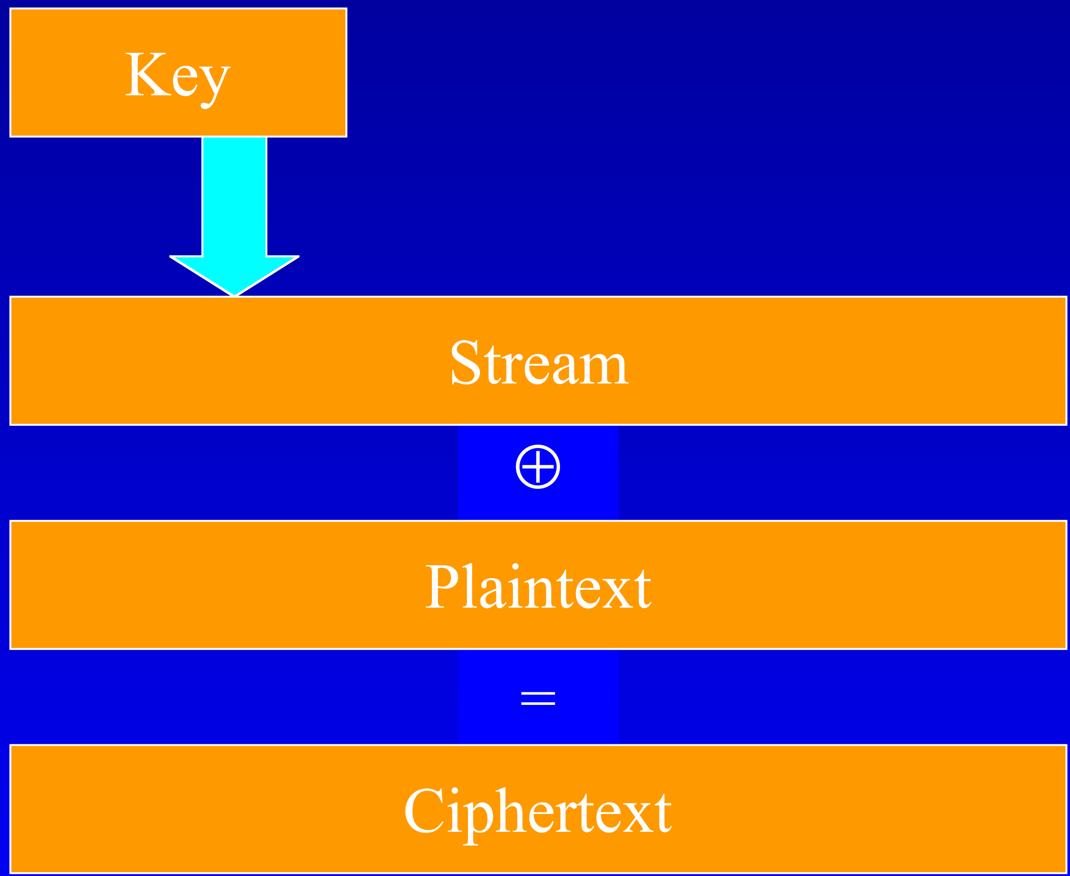
Lecture 2

Symmetric Encryption: Stream & Block Ciphers

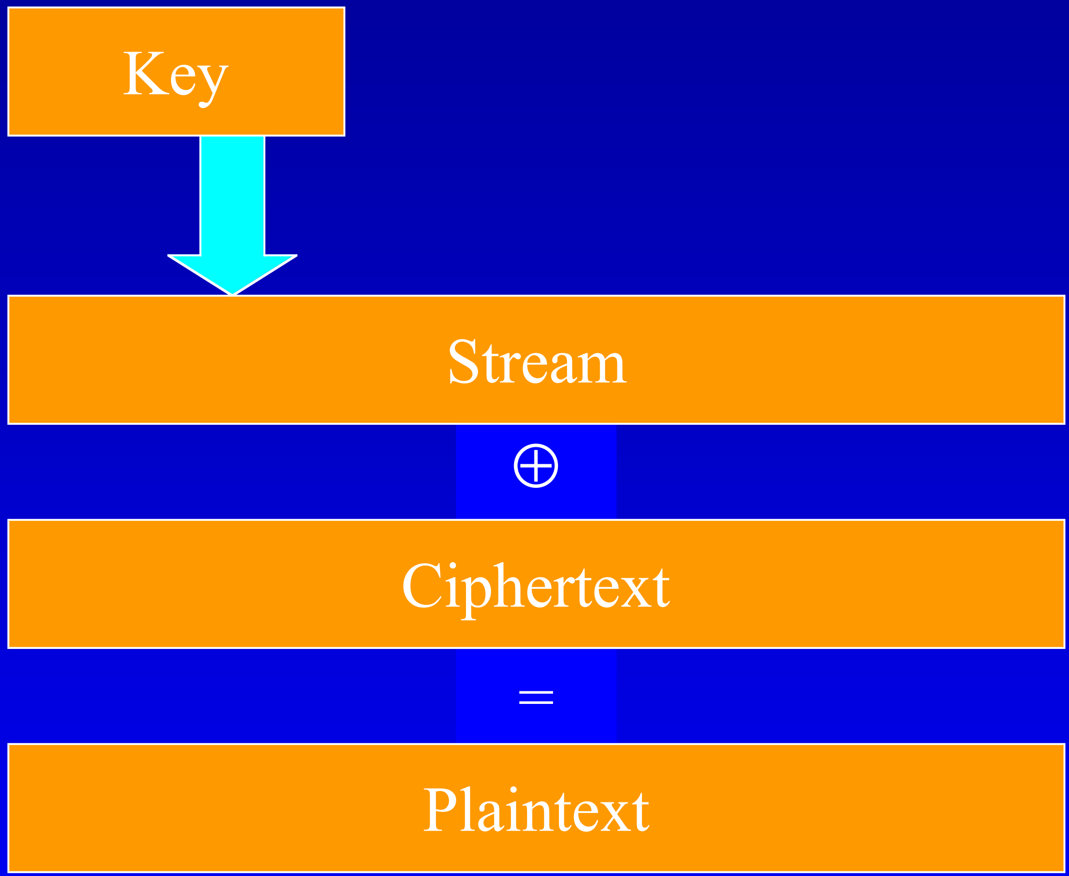
Stream Ciphers

- Start with a secret key (“seed”)
- Generate a keying stream
- i-th bit/byte of keying stream is a function of the key and the first i-1 ciphertext bits.
- Combine the stream with the plaintext to produce the ciphertext (typically by XOR)

Example of Stream Encryption



Example of Stream Decryption



Real Cipher Streams

- Most pre-WWII machines
- German Enigma
- Linear Feedback Shift Register
- A5 – encrypting GSM handset to base station communication
- RC-4 (Ron's Code)

Terminology

Stream cipher is called synchronous if keystream does not depend on the plaintext (depends on key alone).

Otherwise cipher is called asynchronous.

Current Example: RC-4

- Part of the RC family
- Claimed by RSA as their IP
- Between 1987 and 1994 its internal was not revealed – little analytic scrutiny
- Preferred export status
- Code released anonymously on the Internet
- Used in many systems: Lotus Notes, SSL, etc.

RC4 Properties

• Variable key size stream cipher with byte oriented operations.

• Based on using a random looking permutation.

• 8-16 machine operations per output byte.

• Very long cipher period (over 10^{100}).

• Widely believed to be secure. Used for encryption in SSL web protocol.

RC-4 Initialization

1. $j=0$
2. $S_0=0, S_1=1, \ldots, S_{255}=255$
3. Let the key be $k_0,\ldots,k_{255}$ (repeating bits if necessary)
4. For i=0 to 255
   - $j = (j + S_i + k_i) \bmod 256$
   - Swap S_i and S_j

RC-4 Key-stream Creation

Generate an output byte B by:

- $i = (i+1) \bmod 256$
- $j = (j + S_i) \bmod 256$
- Swap S_i and S_j
- $t = (S_i + S_j) \bmod 256$
- $B = S_t$

B is XORed with next plaintext byte

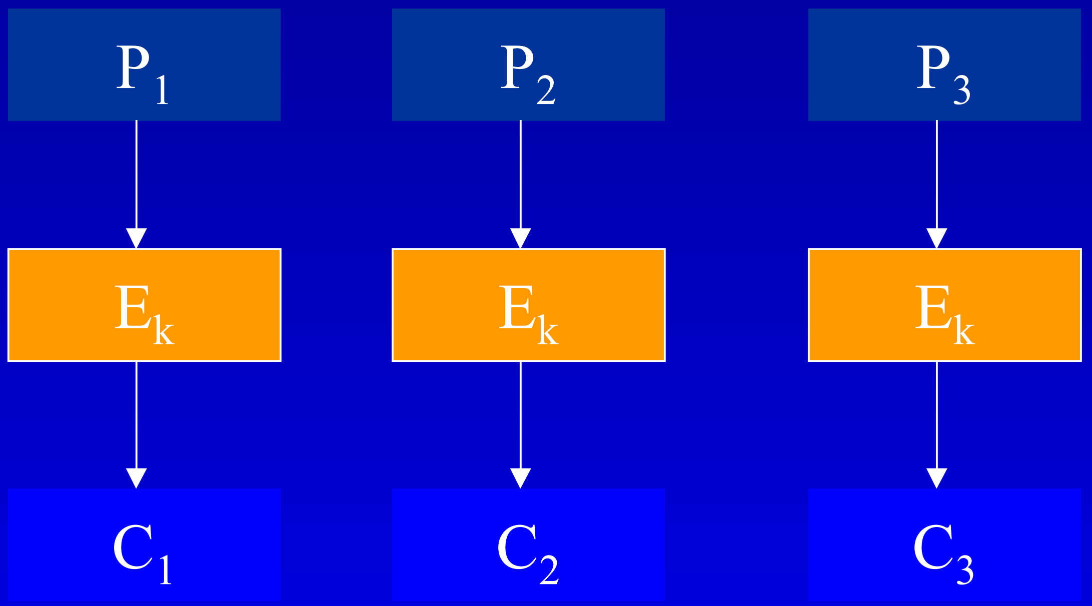
Block Ciphers

- Encrypt a block of input to a block of output
- Typically, the two blocks are of the same length
- Most symmetric key systems block size is 64
- In AES block size is 128
- Different modes for encrypting plaintext longer than a block

Real World Block Ciphers

- DES, 3-DES
- AES (Rijndael)
- RC-2
- RC-5
- IDEA
- Blowfish, Cast
- Gost

ECB Mode Encryption
(Electronic Code Book)



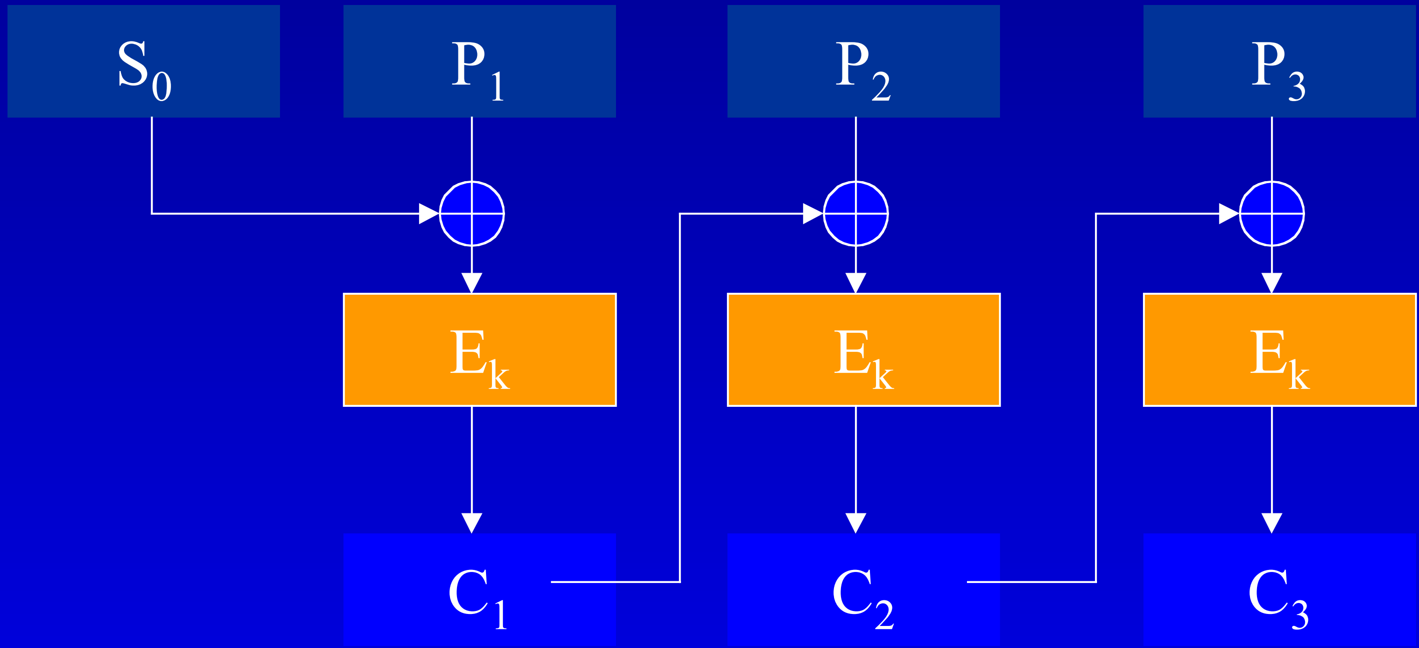
encrypt each plaintext block separately

Properties of ECB

- Simple and efficient
- Parallel implementation possible
- Does not conceal plaintext patterns
- Active attacks are possible (plaintext can be easily manipulated by removing, repeating, or interchanging blocks).

CBC Mode Encryption

(Cipher Block Chaining)



Previous ciphertext is XORed with current plaintext before encrypting current block.

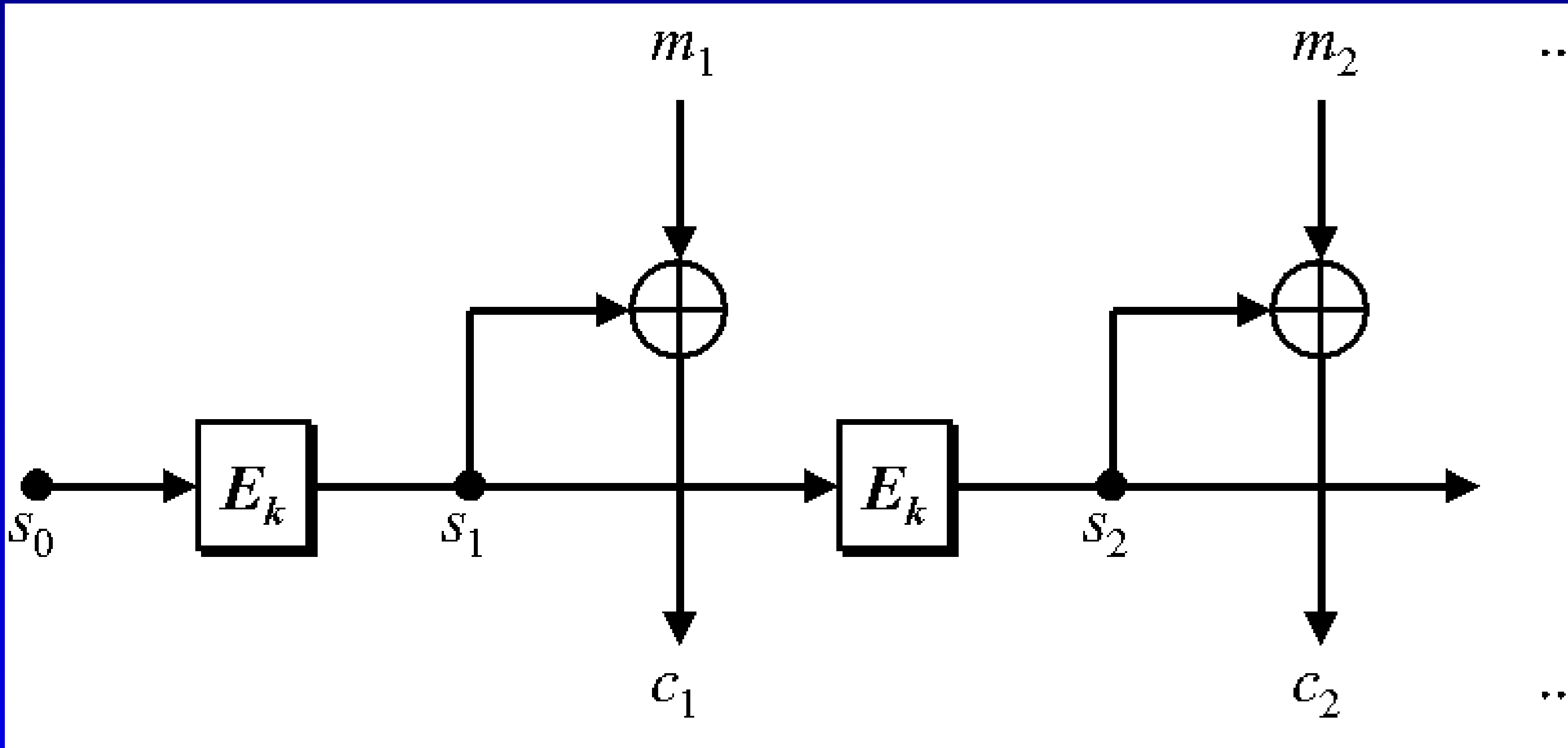
An initialization vector S_0 is used as a “seed” for the process.

Seed can be “openly” transmitted.

Properties of CBC

- Asynchronous stream cipher
- Errors in one ciphertext block propagate
- Conceals plaintext patterns
- No parallel implementation known
- Plaintext cannot be easily manipulated.
- Standard in most systems: SSL, IPSec etc.

OFB Mode
(Output FeedBack)



An initialization vector s_0 is use as a ``seed'' for a sequence of data blocks s_i

Properties of OFB

- Synchronous stream cipher
- Errors in ciphertext do not propagate
- Pre-processing is possible
- Conceals plaintext patterns
- No parallel implementation known
- Active attacks by manipulating plaintext are possible

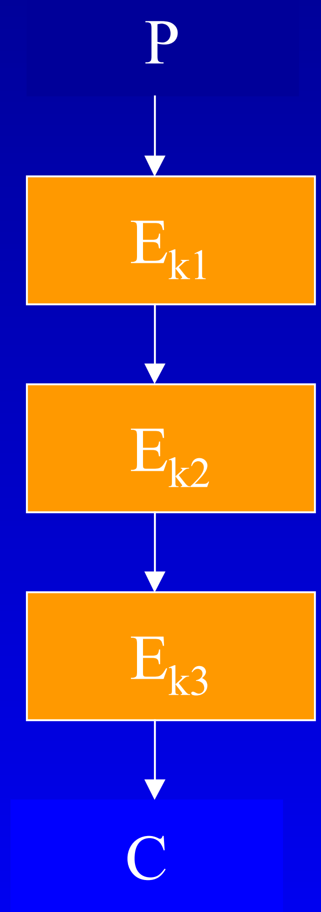
AES Proposed Modes

- CTR (Counter) mode (OFB modification): Parallel implementation, offline pre-processing, provable security, simple and efficient
- OCB (Offset Codebook) mode - parallel implementation, offline preprocessing, provable security (under specific assumptions), authenticity

Strengthening a Given Cipher

- Design multiple key lengths – AES
- Whitening - the DESX idea
- Iterated ciphers – Triple DES (3-DES), triple IDEA and so on

Triple Cipher - Diagram



Iterated Ciphers

- Plaintext undergoes encryption repeatedly by underlying cipher
- Ideally, aach stage uses a different key
- In practice triple cipher is usually

  $C= E_{k1}(E_{k2}(E_{k1}(P)))$ [EEE mode] or

  $C= E_{k1}(D_{k2}(E_{k1}(P)))$ [EDE mode]

  EDE is more common in practice

Necessary Condition

- For some block ciphers iteration does not enhance security
- Example – substitution cipher
- Consider a block cipher: blocks of size b bits, and key of size k
- The number of all possible functions mapping b bits to b bits is $(2^b)^{2^b}$

Necessary Condition (cont.)

- The number of all **possible** encryption functions (bijections) is $2^b!$
- The number of encryption functions in our cipher is at most 2^k.
- Claim: The bijections are a group G under the ° operation (composition)
- Claim: If the encryptions of a cipher form a sub-group of G then iterated cipher does not increases security.

Meet in the Middle Attack

- Double ciphers are rarely used due to this attack
- Attack requires
  - Known plaintext
  - 2^{k+1} encryptions and decryptions
  - $|k|2^{|k|}$ storage space
- A square root of trivial attacking time at the expense of storage

Meet in the Middle (cont.)

- Given a plaintext-ciphertext pair *(p,c)*
  - Compute & store the table of $D_{k2}(c)$ for all k_2 takes 2^k decryptions, $|k|2^{|k|}$ storage.
  - For every k_1, test if $E_{k1}(p)$ is in table
  - Every hit gives a possible k_1,k_2 pair
  - May have to repeat several times
- Meet in the middle is applicable to any iterated cipher, reducing the trivial processing time by 2^k encryptions

Two or Three Keys

- Sometimes only two keys are used in 3-DES
- Identical key must be at beginning and end
- Legal advantage (export license) due to smaller overall key size
- Used as a KEK in the BPI protocol which secures the DOCSIS cable modem standard

Some Group Theory

Sub-groups

- Let $(G,\oplus)$ be a group. $(H,\oplus)$ is a sub-group of $(G,\oplus)$ if it is a group, and $H \subseteq G$
- Claim: If G is finite and $(H,\oplus)$ is closed, then $(H,\oplus)$ is a sub-group of $(G,\oplus)$.
- Examples
- Lagrange theorem: if G is finite and $(H,\oplus)$ is a sub-group of $(G,\oplus)$ then |H| divides |G|

Order of Elements

- Let a^n denote $a \oplus, \ldots, \oplus a$ n times
- We say that a is of order n if $a^n = 1$, and for any $m < n$, $a^m \neq 1$
- Examples
- Euler theorem: in the multiplicative group of Z_n any element is of order at most $\phi(n)$

Adversary's Goals

- Final goal: recover key
- Intermediate goals:
  - Reduce key space
  - Discover plaintext patterns
  - Rec

  - over portions of plaintext
  - Change ciphertext to produce meaningful plaintext, without breaking the system

  (active attack)

Generic Attacks

- Exhaustive search
  - Type: ciphertext only
  - Time: $2^{|k|}$ decryptions per ciphertext
  - Storage: constant
- Table lookup
  - Type: chosen plaintext
  - Time: offline $2^{|k|}$ decryptions, online constant
  - Storage: $2^{|k|}$ ciphertexts