Introduction to Modern Cryptography

Lecture 10

Digital Signatures

Handwritten Signatures

- Relate an individual, through a handwritten signature, to a document.
- Signature can be verified against a prior authenticated one, signed in person.
- Should be hard to forge.
- Are legally binding (convince a third party, e.g. a judge).

Digital Signatures: Desired Properties

- Relate an individual, through a digital string, to a document.
- Signature should be easy to verify.
- Should be hard to forge.
- Are legally binding (convince a third party, e.g. a judge).

Diffie and Hellman (76) "New Directions in Cryptography"

Let E_A be Alice's public encryption key,
and let D_A be Alice's private decryption key.

- To sign the message M, Alice computes the string $y=D_A(M)$ and sends M,y to Bob.
- To verify this is indeed Alice's signature, Bob computes the string $x = E_A(y)$ and checks $x=M$.

Intuition: Only Alice can compute $y=D_A(M)$, thus forgery should be computationally infeasible.

Problems with "Pure" DH Paradigm

- Easy to forge signatures of random messages even without holding D_A:

Bob picks R arbitrarily, computes $S=E_A(R)$.

Then the pair (S,R) is a valid signature

of Alice on the "message" S.

- Therefore the scheme is subject to existential forgery.
- "So what" ?

Problems with “Pure” DH Paradigm

- Consider specifically RSA. Being multiplicative, we have (products mod N)

$D_A(M_1M_2) = D_A(M_1)\, D_A(M_2)$.

- If M_2=“I OWE BOB $20” and M_1=“100” then under certain encoding of letters we could get M_1M_2 =“I OWE BOB $2000”…

Standard Solution: Hash First

Let E_A be Alice's public encryption key, and let D_A be Alice's private decryption key.

- To sign the message M, Alice first computes the strings $y=H(M)$ and $z=D_A(y)$. Sends M,z to Bob.
- To verify this is indeed Alice's signature, Bob computes the string $y=E_A(z)$ and checks $y=H(M)$.

- The function H should be collision resistent, so that cannot find another M' with $H(M)=H(M')$.

General Structure: Signature Schemes

- Generation of private and public keys (randomized).
- Signing (either deterministic or randomized)
- Verification (accept/reject) - usually deterministic.

Schemes Used in Practice

- RSA
- El-Gamal Signature Scheme (85)
- The DSS (digital signature standard, adopted by NIST in 94 is based on a modification of El-Gamal signature.

El-Gamal Signature Scheme

Generation

- Pick a prime p of length 1024 bits such that DL in Z_p^* is hard.
- Let g be a generator of Z_p^*.
- Pick x in $[2,p-2]$ at random.
- Compute $y=g^x \bmod p$.
- Public key: p,g,y.
- Private key: x.

El-Gamal Signature Scheme

Signing M

- Hash: Let $m=H(M)$.
- Pick k in $[1,p-2]$ relatively prime to $p-1$ at random.
- Compute $r=g^k \bmod p$.
- Compute $s=(m-rx)k^{-1} \bmod (p-1)$ (***)
- Output r and s.

El-Gamal Signature Scheme

Verify M,r,s,PK

- Compute $m=H(M)$.
- Accept if $0<r<p$ and $y^r r^s = g^m$ mod p.

  else reject.
- What's going on?

By (***) $s=(m-rx)k^{-1}$ mod $p-1$, so $sk+rx=m$. Now $r=g^k$ so $r^s=g^{ks}$, and $y=g^x$ so $y^r=g^{rx}$, implying $y^r r^s=g^m$.

Signatures vs. MACs

Suppose parties A and B share the secret key K. Then M, $MAC_K(M)$ convinces A that indeed M originated with B. But in case of dispute A cannot convince a judge that M, $MAC_K(M)$ was sent by B, since A could generate it herself.