# Problem #15

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*Summary: Is the extension of Combinatory Logic by Boolean constants confluent?*

Consider the following extension of Combinatory Logic with constants $T$ (true), $F$ (false), $C$ (conditional):

$$\begin{array}{rcl} Ix & \to & x \\ Kxy & \to & x \\ Sxyz & \to & (xz)(yz) \\ CTxy & \to & x \\ CFxy & \to & y \\ x \leftrightarrow^* y \Rightarrow Czxy & \to & x \end{array}$$

Is this (non-terminating) "semi-equational" (or "natural", as such are called in [DO90]) conditional rewrite system confluent? Note that if we take the above system plus the rule $x \leftrightarrow^* y \Rightarrow Czxy \to y$, the resulting conditional rewrite system *is* confluent (cf. [Klo92][de 90]).

# Bibliography


[de 90] Roel C. de Vrijer. Unique normal forms for Combinatory Logic with parallel conditional, a case study in conditional rewriting. Technical report, Free University, Amsterdam, 1990.

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[Klo92] Jan Willem Klop. Term rewriting systems. In S. Abramsky, D. M. Gabbay, and T. S. E. Maibaum, editors, *Handbook of Logic in Computer Science*, volume 2, chapter 1, pages 1–117. Oxford University Press, Oxford, 1992.