Problem #15

Originator: Yoshihito Toyama
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Summary: Is the extension of Combinatory Logic by Boolean constants confluent?

Consider the following extension of Combinatory Logic with constants T (true), F (false), C (conditional):

$$
\begin{aligned}
Ix &\rightarrow x \\
Kxy &\rightarrow x \\
Sxyz &\rightarrow (xz)(yz) \\
CTxy &\rightarrow x \\
CFxy &\rightarrow y \\
x \leftrightarrow^* y \Rightarrow Czxy &\rightarrow x
\end{aligned}
$$

Is this (non-terminating) "semi-equational" (or "natural", as such are called in [DO90]) conditional rewrite system confluent? Note that if we take the above system plus the rule $x \leftrightarrow^* y \Rightarrow Czxy \rightarrow y$, the resulting conditional rewrite system *is* confluent (cf. [Klo92][de 90]).

Bibliography

[de 90] Roel C. de Vrijer. Unique normal forms for Combinatory Logic with parallel conditional, a case study in conditional rewriting. Technical report, Free University, Amsterdam, 1990.

[DO90] Nachum Dershowitz and Mitsuhiro Okada. A rationale for conditional equational programming. *Theoretical Computer Science*, 75:111–138, 1990.

[Klo92] Jan Willem Klop. Term rewriting systems. In S. Abramsky, D. M. Gabbay, and T. S. E. Maibaum, editors, *Handbook of Logic in Computer Science*, volume 2, chapter 1, pages 1–117. Oxford University Press, Oxford, 1992.

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