Analysis of Algorithms

Problem Set no. 2 — Minimum directed spanning trees

Given: May 5, 2009

Exercise 1.1 Exhibit a directed graph in which the lightest edge participates in a directed spanning tree rooted at some vertex \( r \), but not in a minimum directed spanning tree rooted at \( r \).

Exercise 1.2 Exhibit a directed graph in which all minimum directed spanning trees rooted at some vertex \( r \) contain the heaviest edge in the graph, even through there are directed spanning trees rooted at \( r \) that avoid that edge.

Exercise 1.3 Exhibit a directed graph \( G = (V, E) \) with two weight functions \( w_1, w_2 : E \to R \) on it such that for every two edges \( e_1, e_2 \in E \) we have \( w_1(e_1) \leq w_1(e_2) \) if and only if \( w_2(e_1) \leq w_2(e_2) \) and yet the minimum directed spanning trees rooted at some vertex \( r \) with respect to \( w_1 \) and \( w_2 \) are different.

Exercise 1.4 Describe a deterministic linear time algorithm for finding a minimum directed spanning tree in an acyclic graph.

Exercise 1.5 Give simple linear time reductions between the problems of finding a minimum directed spanning tree rooted at a given vertex \( r \), and the problem of finding a minimum directed spanning tree rooted at an arbitrary vertex of the graph.

Exercise 1.6 Let \( G = (V, E) \) be a directed graph with a weight function \( w : E \to R \) defined on its edges. A branching \( B \) of \( G \) is a set of edges \( B \subseteq E \) such that the subgraph \( (V, B) \) is acyclic and the indegree of each vertex in it is at most 1. Obtain an efficient algorithm for finding a maximum branching of \( G \), i.e., a branching \( B \) for which \( \sum_{e \in B} w(e) \) is maximized. (Note that some of the edge weights may be negative.)