Lecture notes for "Analysis of Algorithms":

Dynamic All-Pairs Shortest Paths

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Slightly updated code

Function `apsp`$(G = (V, E, w))$

```
Q ← make-heap()
foreach v ∈ V do
    π[v] ← make-path(v)
foreach u, v ∈ V do
    P[u, v] ← make-heap()
foreach v ∈ V do
    insert-in-edges(G, v)
build-paths(G)
```

Function `insert-in-edges`(G, v)

```
foreach e = (u, v) ∈ E_in[v] do
    π ← make-path(e)
    insert(L[π[v]], π)
    insert(R[π[u]], π)
    heap-insert(P[u, v], π)
```

Function `insert-out-edges`(G, v)

```
foreach e = (v, u) ∈ E_out[v] do
    ⋮
```

Function `build-paths`(G)

```
foreach u, v ∈ V do
    sp[u, v] ← null
    if P[u, v] ≠ φ then
        π ← find-min(P[u, v])
        heap-insert(Q, π)
while Q ≠ φ do
    π ← extract-min(Q)
    u ← start[π] ; v ← end[π]
    if sp[u, v] = null then
        sp[u, v] ← π
        if ¬sel[π] then
            new-shortest-path(π)
```

Function `new-shortest-path`(π)

```
sel[π] ← true
insert(L*[r[π]], π)
insert(R*[l[π]], π)
foreach π' ∈ L*[l[π]] do
    π'' ← make-path(π', π)
    insert(L[π], π'')
    insert(R[π'], π'')
    u ← start[π''] ; v ← end[π'']
    heap-insert(P[u, v], π'')
    heap-insert(Q, π'')
foreach π' ∈ R*[r[π]] do
    ⋮
```

```
Function remove-paths(G, v)
    S ← make-stack()
    push(S, π[v])
    while S ≠ φ do
        π ← pop(S)
        foreach π' ∈ L[π] ∪ R[π] do
            push(S, π')
            delete(L[r[π']], π')
            delete(R[l[π']], π')
            u ← start[π'] ; v ← end[π']
            heap-delete(P[u, v], π')
            if sel[π'] then
                delete(L*[r[π']], π')
                delete(R*[l[π']], π')
```

```
Function update(G, v)
    remove-paths(G, v)
    insert-in-edges(G, v)
    insert-out-edges(G, v)
    build-paths(G)
```

```
Function full-update(G, v)
    v[t] ← v
    update(G, v)
    j ← 1
    while j ≤ t do
        update(G, v[t − j])
        j ← 2j
    t ← t + 1
```

References

[DI04] C. Demetrescu and G.F. Italiano. A new approach to dynamic all pairs shortest paths. *Journal of the ACM*, 51(6):968–992, 2004.