Problem Set no. 2

Given: June 5, 2007
Due: June 19, 2007

Exercise 2.1 Find a graph on n vertices that has $\binom{n}{2}$ global minimum cuts.

Exercise 2.2 Consider the following algorithm for finding a minimum global cut in an unweighted undirected graph on n vertices: Choose a random edge and contract it. Repeat this operation until the number of remaining vertices in the graph is t, where t is a parameter to be determined later. Then, apply a deterministic algorithm for finding a global minimum cut in the resulting graph. The complexity of the deterministic minimum cut algorithm is $O(n^3)$, where n is the number vertices in the graph. Finally, run this combined algorithm a sufficient number of times so that the probability that it finds a global minimum cut is at least $1/2$. What is the optimal choice of t and what is then the running time of this algorithm?

Exercise 2.3 Show that a simple variant of the random contraction algorithm of Karger and Stein can be used to find a *minimum 3-cut* of an undirected and unweighted graph $G = (V, E)$ on n vertices. (A 3-cut of $G = (V, E)$ is a partition of V into three non-empty sets A, B and C. The size of the cut is the number of edges connecting vertices from different sets.) What is the success probability of the algorithm?

Exercise 2.4 Prove Dilworth's theorem: Any *partial order* $(R, \preceq)$ on k elements contains either a *chain* or an *anti-chain* of at least $\sqrt{k}$ elements. A chain is a totally ordered subset of R. An anti-chain is a subset of R in which every two elements are incomparable.)

Exercise 2.5 Let $G = (V, E)$ be a directed graph, with $|V| = n$ and $|E| = m$, and $w : E \to \{-N, \ldots, 0, \ldots, N\}$. Describe an algorithm that finds a minimum mean-weight cycle in G. Your algorithm should be as efficient as possible.