CS277: CONCRETE COMPLEXITY

TAKE-HOME EXAM

given: December 12, 1996
due: December 17, 1996

Answer **three** of the following four questions.

Question 1

Derive a Nečiporuk-type lower bound for the size of directed contact networks. In other words, obtain a lower bound on the size of any contact network computing a function f in terms of $r_{V_i}(f)$, the number of different restrictions the function f has, where $V_1, \ldots, V_p$ is a partition of the variable set of f. Make your lower bound as large as you can.

What is the highest lower bound that can be obtained using your Nečiporuk bound for contact networks? Describe a sequence of functions for which this highest lower bound is obtained.

Question 2

Show that the MAJORITY function is not in AC^0 using the following two methods: (1) Show that $MAJORITY \in AC^0$ implies $PARITY \in AC^0$. (2) Give a direct proof using Håstad's switching lemma.

Question 3

Obtain an almost tight lower bound on the monotone complexity of $CLIQUE_4(n)$.

Question 4

Describe the construction of an ϵ-nearsorter using ϵ'-halvers as basic building blocks. What is the depth of your construction? Prove that the network constructed is indeed an ϵ-nearsorter. (An *ϵ-nearsorter* is a network with n inputs and n outputs such that for every $k \leq n/2$, at most ϵk of the k smallest input elements do not reach one of the first $k + \epsilon n$ output wires. Similarly, for every $k \leq n/2$, at most ϵk of the k largest input elements do not reach one of the last $k + \epsilon n$ output wires.)