Problem Set no. 5

Given: January 23, 2020

Exercise 5.1 Consider the separating automaton of [CDFJLP (2018)]. (a) Let $w \in AllCycEven_{n,d}$. Derive an upper bound on the number of steps after which the separating automaton accepts w. (b) Let $w \in LimsupEven_d$. Does the automaton necessarily accepts w? After how many steps?

Exercise 5.2 Let $G = (V, E, p)$, where $p : E \to [d]$, be an even graph, i.e., a graph in which the maximum priority on every cycle is even. Assume that d is even. Let L_i be the set of vertices from which there is a path containing i edges of priority $d-1$ and no edges of priority d, but there is no such path containing $i+1$ edges of priority $d-1$. Prove that: (1) $V = \cup_{i=0}^{n-1} L_i$. (2) If $(u, v) \in E$, $p(u, v) = d-1$, $u \in L_j$, $v \in L_i$, then $j > i$. (3) If $(u, v) \in E$, $p(u, v) \leq d-1$, $u \in L_j$, $v \in L_i$, then $j \geq i$.

Exercise 5.3 Describe a polynomial time algorithm that given an even graph $G = (V, E, p)$, where $p : E \to [d]$ and d even, finds a progress measure $f : V \to [0, n-1]^{d/2} \cup \{\infty\}$. What is the complexity of the algorithm?

Exercise 5.4 Let $G = (V = V_0 \cup V_1, E, p)$, where $p : E \to [d]$ and d even, be a parity game. Let $f^*(u) = \min\{f(u) \mid f \text{ is a progress measure of } G\}$, for every $u \in V$. (a) Prove that f^* is a progress measure. (b) Prove that for every $u \in V$, EVEN can win from u if and only if $f^*(u) < \infty$.

Exercise 5.5 Prove that a maximal even (n, d)-multigraph is tree-like. (See slide 57 of Lecture 6.)

Exercise 5.6 Let $G = (V, E)$ be a tree-like (n, d)-multigraph, where $E \subseteq V \times [0, d] \times V$. Let $E_p = \{(u, v) \in V \times V \mid (u, p, v \in E)$. Show that each equivalence class of E_p, where $p > 0$ is even, is composed of a disjoint union of equivalence classes of E_{p-2}. (See slide 59 of Lecture 6.)

Exercise 5.7 Prove that a tree t is an ordered subtree of tree Tif and only if there is a homomorphism from $G(t)$ to $G(T)$.