Wavelet packet based algorithm for identification of quasi-periodic signals

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ABSTRACT

We present a generic approach that identifies and differentiates among signals of wide range of problems. Originally our algorithm was developed to detect the presence of a specific vehicle belonging to a certain class via the analysis of the acoustic signals emitted while it is moving. A crucial factor in having a successful detection (no false alarm) is to construct signatures built from characteristic features that enable to discriminate between the class of interest and the residual information such as background. We construct the signatures of certain classes by the distribution of the energies among blocks which consist of wavelet packet coefficients. We developed an efficient procedure for adaptive selection of the characteristic blocks. We modified the CART algorithm in order to utilize it to be a decision unit in our scheme. However, this technology, which has many algorithmic variations, can be used to solve a wide range of classification and detection problems which are based on acoustic processing and, more generally, for classification and detection of signals which have near-periodic structure. We present results of successful application of the properly modified algorithm to detection of early symptoms of arterial hypertension in children via real-time analysis of pulse signals.

1. INTRODUCTION

A number of experimental evidence indicate that biomedical signals from the human body contain a significant diagnostic information. Oscillations in the human body are generated by higher hierarchical levels of the control system. Because of superposition of oscillations of different levels, the structure of the biomedical signals detected in the human body is pretty complicated but at the same time bears a lot of information on the inner processes. These signals are superpositions of various oscillating components of different amplitude, phase and frequency originated from various sources inside the body. All aforesaid is completely related to the pulse signals.

We present a preliminary approach to solving the problem of identification of the pulse signals of the radial artery. The goal of the research is to explore possibilities of the automatic diagnostics of various diseases via real-time analysis of the rhythmic structure of the pulse signals. In this stage of the investigation we were concerned with the distinguishing of the pulse signals taken from the children and adolescents (ages 9 to 15 years) with early symptoms of the hypertension from signals related to other diseases. Solution of this problem gives way to a reliable non-invasive diagnostics of the disease from the recording of the pulse signal. The problem is especially complex since even the state of a single person can vary significantly, to set aside a great variability of persons manifesting the symptoms of the disease. Moreover, we analyzed the recordings related to *early* symptoms of the hypertension which are hardly detected even by conventional methods.

We used a generic approach to solving a wide class of identification problems concerned with the signals whose structure is quasi-periodic. Actually, the we employed a modified version of the algorithm which originally was developed for solving the problem of detection of the presence of a vehicle belonging to a certain class via the analysis of the acoustic signals emitted while it is moving Ref. 1.

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A crucial factor in having a successful detection (no false alarm) is to construct signatures built from characteristic features that enable to discriminate between the class of interest and the residual.

The basic assumption is that the signature for the class of pulse signals related to a certain disease is obtained as a combination of the inherent energies in a small set of the most discriminant blocks of the wavelet packet coefficients of the signals (Ref. 6). It is justified by the fact that contribution of each source of oscillations inside the human body in frequency domain contains only a few dominating bands. As the conditions are changed the configuration of these bands may vary, but the general disposition remains. Therefore, the blocks of the wavelet packet coefficients, each of which is related to a certain frequency band, are the relevant tool to base the classification on. These blocks contain the distinctive characteristic features.

In the final phase of the process, in order to identify the acoustic signatures of the presented signals, we used two conventional classifiers: the properly modified Classification and Regression Tree (CART) Ref. 2, Ref. 1 and Linear Discriminant Analysis Ref. 5. We apply also a classifier which is some combination of the two former ones. We call it FUSION.

The obtained results demonstrate that the developed algorithm is robust and the false alarm rate is near zero. Thus it can serve as a base for the development of a flexible diagnostic system.

However, the signals with similar quasi-periodic structure appear in many biomedical identification problems and we anticipate that the algorithm (with proper modifications) will be relevant to solving these problems.

2. FORMULATION OF THE APPROACH

2.1. Structure of the signals.

We assume that the recorded pulse signals belong to one of the two classes $C^k,\ k = 0, 1$. To Class C^1 we assign the signals taken from the persons with confirmed by other methods symptoms of hypertension. To Class C^0 we assign the signals taken from persons suffering from other disfunctions, such as nocturnal enuresis, chronic subfebrilitet condition, cephalalgia etc. The signals were sampled at 100 Hz.

Figure 1 shows some portions of the signals belonging to Class C^1 taken from different persons and their Fourier transforms. Figure 2 does the same for Class C^2.

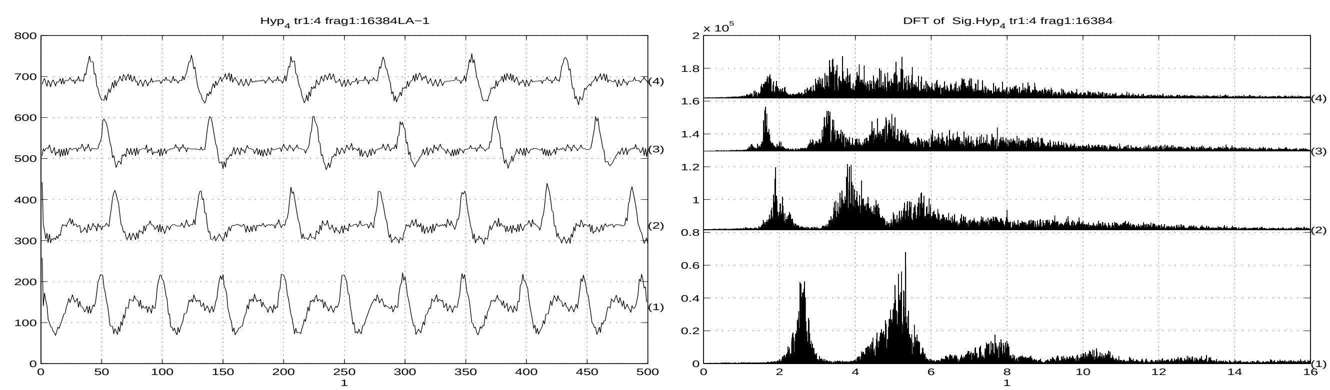

Figure 1. Left picture: portions of Class C^1-signals. Right picture: portions of the Fourier transforms of the signals shown in the left picture.

We realized that even within the same class the signals differ significantly from each other. The same is true for their spectra. However, there are some common properties to all the pulse signals that were recorded from various persons. First, these signals are quasi-periodic in the sense that there exist some dominating frequencies in each signal. These frequencies may vary from one person to another. However, for the persons which acquired the same disease the variations of some frequencies are confined in narrow frequency bands. Moreover, the relative locations of these frequency bands are stable (invariant) to some extent for signals that belong to Class C^1 and differ for signals from another class.

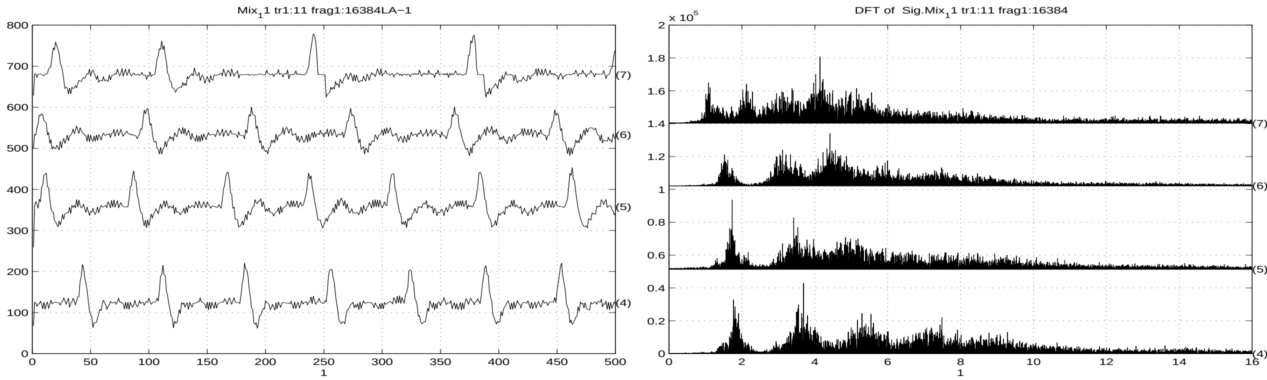

Figure 2. Left picture: portions of Class C^2-signals. Right picture: portions of the Fourier transforms of the signals shown in the left picture.

Therefore, we conjectured that the distribution of the energy (or some energy-like parameters) of signals that belong to Class C^1 over different areas of the frequency domain may provide a reliable characteristic signature for this class.

2.2. Formulation of the approach

Wavelet packet analysis (Ref. 6) is a highly relevant tool for adaptive search for valuable frequency bands of a signal or a class of signals. Once implemented, the wavelet packet transform of a signal yields a huge variety of different partitions of the frequency domain. One of the important features of the transform is its computational efficiency. The implementation of an m level transform requires $O(mn)$ operations. Due to the lack of time invariance in the multiscale wavelet packet decomposition, we will deal with the whole blocks of wavelet packet transform rather than with individual coefficients and waveforms. Moreover, we increase the number of sample signals in the training sets and in the test sets by imposing a comparatively short window on each input signal followed by a shift of this window along the signal so that adjacent sections have some overlap.

1. For training we use a set of signals with known membership. From them we select a few blocks which discriminate efficiently between the given classes of signals.
2. We apply the wavelet packet transform on the signal to be classified. We use as its characteristic features the normalized l_2 or l_1 norms of the wavelet packet coefficients contained in the selected blocks.
3. Finally, we submit the vectors of the extracted features to the modified CART or(and) LDA classifiers. The latter, being appropriately trained beforehand, decide which class this signal belongs to.

The algorithm is centered around two basic issues:

Selection of the discriminant blocks of the wavelet packet coefficients is done according to the following steps:

1. Choice of the analyzing filters.
2. Construction of the training set.
3. Calculation of the energy map.
4. Evaluation of the discriminant power of the decomposition blocks.
5. Selection of the discriminant blocks.

Discrimination among the signals:

1. Preparation of the pattern set.

2. Building the CART classification trees.
3. Preparation of the testing set.
4. Making the decision.

3. IMPLEMENTATION

3.1. Selection of discriminant blocks

Choice of analyzing waveforms: We use 8-th order spline wavelet packets. These filters reduce the overlapping among frequency bands associated with different decomposition blocks. The empirical analysis suggests to decompose the signals into 7 to 8 levels (scales).

Construction of the training set: Initially, we gather as many recordings as possible for each class $C^l,\ l = 0,\, 1$ which have to be separated. Then we prepare from each selected recording, which belongs to a certain class, a number of overlapping slices of length $n = 2^J$ samples each, shifted with respect to each other by $s \ll n$ samples. All together we produce M^l slices for the class C^l. These groups of slices form the training set for the search of discriminant blocks.

Calculation of the energy map: First, we specified the kind of energy type measure to be used. Typically, we use either the normalized l_2 or l_1 norms of the blocks. After the measure has been chosen, the following operations are carried out:

- The wavelet packet transform is applied up to scale m on each slice of length n from a given class C^l. This procedure produces mn coefficients arranged into $2^{m+1} - 1$ blocks associated with different frequency bands.
- The slice $A^l(i, :)$ is decomposed. The "energies" of each block are calculated in accordance with the chosen measure. As a result we obtain, the distribution of the "energies" of the slice $A^l(i, :)$ over various frequency bands of widths from $N_F/2$ to N_F/m, where N_F is Nyquist frequency. It is presented by an energy vector E_i^l of length $2^{m+1} - 1$.
- The energy vectors along the training set of the class are averaged: $E^l = \frac{1}{M} \sum_{i=1}^{M} E_i^l$. The average energy map E^l of length $2^{m+1}-1$ indicates how the distribution of the "energies" among various block of the decomposition and frequency bands, respectively, is taking place within the whole class C^l. Similar operations are performed on both classes C^l, , $l = 0,\, 1$.

Evaluation of the discriminant power of decomposition blocks The average energy map E^l yields some sort of characterization for the class C^l but it is highly redundant. To gain a more concise and meaningful representation of the class we select the most discriminating blocks. One possible way to do so is the following. First, note that for a two-class problem the difference between two maps provides some insight into the problem. In our scheme the term-wise difference (absolute values) of the energy maps serves as the discriminant power map for the decomposition blocks $|E^1 - E^2|$.

Selection of discriminating blocks Now we are in a position to select a few discriminant blocks which form a sort of signatures for the classes. To avoid the frequency overlap we apply the procedure somewhat similar to the *Best Basis Selection Algorithm* Ref. 3. As a result we obtain some non-overlapping set of blocks which map the whole frequency domain of our signals, which are referred to as the "most discriminating basis". We select from this set the blocks with the highest discriminant factor. Moreover, if we are interested in certain frequency bands, we can select the corresponding blocks.

Conclusion: As a result of the operations described above we discover a relatively small set of decomposition blocks such that the distribution of the energies amongst them characterize the classes to be distinguished. This part of the investigation is computationally expensive. On the other hand, this task is performed once.

3.2. Classification

Once we have the set of discriminant blocks $B_1, ..., B_t$, we proceed to the classification phase.

Preparation of the reference set. Initially, we chose a number of recordings that belong to the classes C^l, $l = 0, 1$, to be distinguished, from which we form the reference set. These recordings are sliced similarly to that which was used for the preparation of the training set. For a certain class C^l, we form from each selected recording from the class a number of overlapping slices each of length n. These are shifted with respect to each other by s samples. We assume that there are μ^l slices related to the class C^l that are are gathered into $\mu^l \times n$ matrix a^l, $l = 0, 1$. Then, we apply the wavelet packet transform up to level m on each row $a^l(i, :)$ of this matrix. After the decomposition of the slice $a^l(i, :)$, we calculate only the "energies" of the t blocks $B_1, ..., B_t$ that were selected before. In doing so we obtain the $1 \times t$ vector $V^l(i, :)$ which we regard as a representative of the slice $a^l(i, :)$. The vectors $V^l(i, :)$ form the $\mu^l \times t$ reference matrix V^l of the class C^l. We do the same for both classes C^l.

The reference sets are used immediately for the LDA classifier and as training sets for building CART. After the construction of the classification tree, we are in a position to classify test signals. To do so we must preprocess these signals.

Preparation of the test set. Suppose we are given a signal S whose membership in a certain class has to be established. The algorithm has to be capable to process either a fragment of the recording, or the entire recording, or even a number of recordings. Acting similarly to the previous stage, we obtain from each slice of the signal the $1 \times t$ vector $W(i, :)$ which we regard as a representative of the slice $T(i, :)$. The vectors $W(i, :)$ form the $K \times t$ test matrix W associated with the signal S.

Making the decision. Once the test matrix W is ready, we present each row $W(i, :)$ of the matrix to the CART or(and) LDA classifier. CART uses the tree that was constructed before on the basis of the pattern sets. Once a vector is presented to the tree it is assigned to one of the subsets X_k of the input space X. This determines the most probable membership of the vector. Then we count the numbers of vectors $W(i, :)$ attributed to each class C^l and make the decision in favor of the class which gets the majority of the vectors $W(i, :)$. The robustness of the decision is checked by the percentage of the vectors $W(i, :)$ attributed to this class.

4. THE RESULTS

We conducted the series of experiments to detect early symptoms of the hypertension in the pulse recordings. We had at our disposal 28 recordings with confirmed hypertension (H-signals), 27 recordings with nocturnal enuresis (E-signals) and 8 recordings related to other diseases such as asthma, chronic subfebrilitet condition, cephalalgia etc (O-signals). The signals were taken by the optoelectronic sensor of the radial artery pulse within 200 sec. and sampled at the rate 100 Hz. Some signals were low-pass filtered.

The best results were achieved with wavelet packets based on splines of order 8. The signals were decomposed up to 8th scale. For the choice of discriminant blocks, formation of the reference set and building CART we used 14 H-signals and 14 E-signals. Then the algorithm was tested on the whole set of available signals. All the signals were tested for the symptoms of the hypertension.

The decisions were made in the three-fold way. As the basic decision unit we used the modified CART algorithm Ref. 1. In parallel the signals were tested by the LDA algorithm which displayed robustness inferior to CART. The third decision unit which we call FUSION. Briefly, the idea is that in the cases when the answers produced by CART were not sufficiently reliable, they were corrected by the LDA.

High diversity in the structure of Class C^2 signals could result in a high rate of misclassification (false alarms). To reduce this rate during the selection process of discriminant blocks we choose blocks whose "energy" characterize Class C^1 signals. Moreover, we modified the CART algorithm so that, in order for a signal to be a member of Class C^1, it has to obey strict conditions that we imposed upon the classifier. These modifications proved to be very efficient. The false alarm rate was reduced to almost zero, whereas by implementation of the standard CART it reached up to 64%.

We display the results of the experiments in a few figures. Each asterisk on the figures correspond to a result of testing a certain recording. The indices of recording are displayed on $x-$axes of the figures. The height of each asterisk (between 0 and 1) means the percentage of the vectors $W(i, :)$ attributed to Class C^1. The green asterisks and connecting lines correspond to the CART answers, the blue ones – to the LDA answers and the red asterisks and lines correspond to the FUSION answers.

Moreover, under each diagram we depict the *"loudness"* level of each recording $S^n = \{s_k^n\}_{k=1}^{20000}$ expressed through root mean square

$$RMS = \sqrt{\sum_{k=1}^{20000} (s_k^n)^2}.$$

In Figure 3 we display results of testing of 14 H-signals which were used in the training stage. One can observe that generally all three classifier produced remarkable robust results. CART and FUSION misclassified two signals, so did LDA but with other cases. In Figure 4 we display results of testing of 10 H-signals which did not participate in

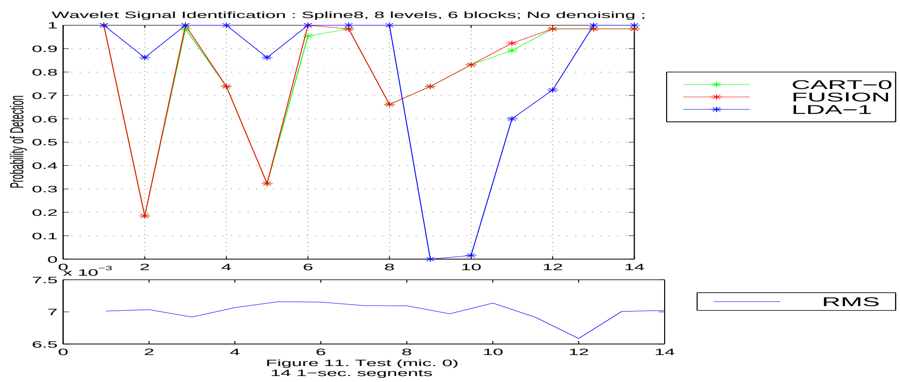

Figure 3. Results of testing of 14 H-signals which were used in the training stage.

the training stage. Again the results are satisfactory. CART and FUSION classified correctly all signals, and LDA misclassified only one case. All above signals were low-pass filtered. In Figure 5 we display results of testing of 5

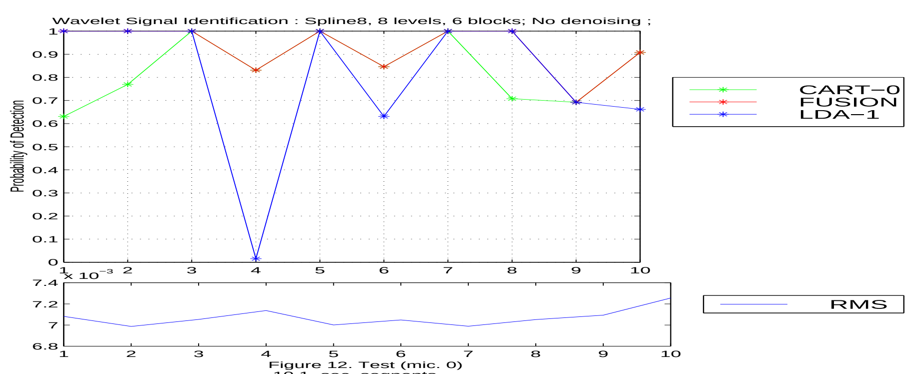

Figure 4. Results of testing of 10 H-signals which did not participate in the training stage.

H-signals which were not subjected to filtering and did not participate in the training stage. Despite the difference with the previous signals in the frequency content, CART and FUSION classified correctly four signals of five, unlike LDA which failed in three cases. In the forthcoming figures we illustrate testing the non-H-signals for the membership in Class C^1. In other words, it is testing of the false alarm rate. So, the lower an asterisk is located, the more correct the corresponding answer is.

In Figure 6 we display results of testing of 14 E-signals which were used in the training stage. CART and FUSION produced not a single false alarm and the answers are highly robust. This is not the case for LDA. In Figure 7 we

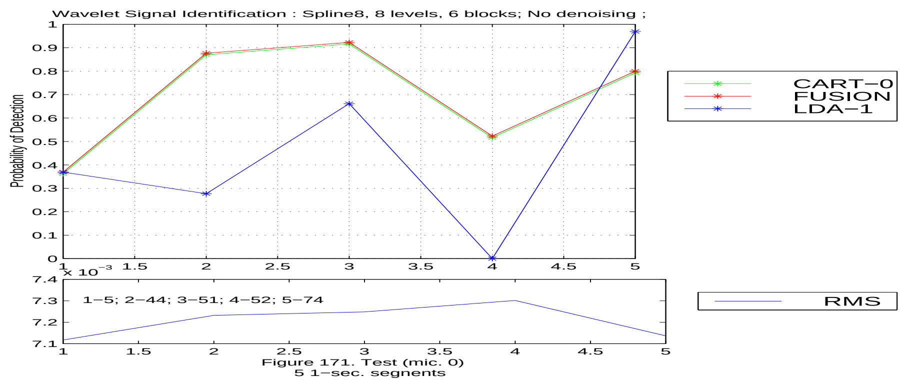

Figure 5. Results of testing of 5 non-filtered H-signals which did not participate in the training stage.

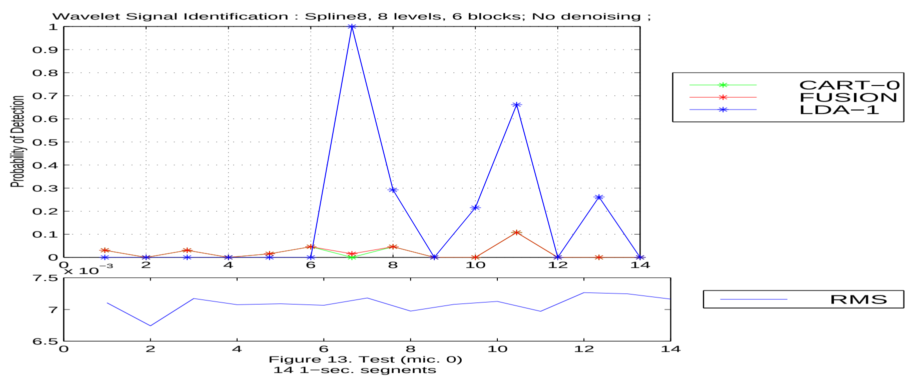

Figure 6. Results of testing of 14 E-signals which were used in the training stage.

display results of testing of 14 E-signals which did not participate in the training stage. Here CART outperformed other classifiers and produced robust correct answers in all cases. FUSION mistook once, but LDA generally failed. In Figure 8 we display results of testing of 9 O-signals which did not participate in the training stage. This case

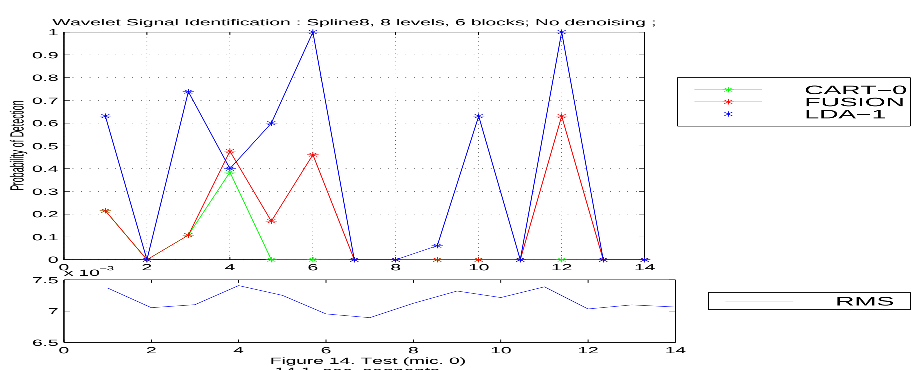

Figure 7. Results of testing of 14 E-signals which did not participate in the training stage.

was more hard for the classification because the choice of the discriminant blocks and training the classifiers were made using only the signals related to the hypertension and enuresis symptoms. But the O-signals correspond to just different diseases. Nevertheless, CART and FUSION produced false alarm only for three signals out of nine.

LDA performed much worse.

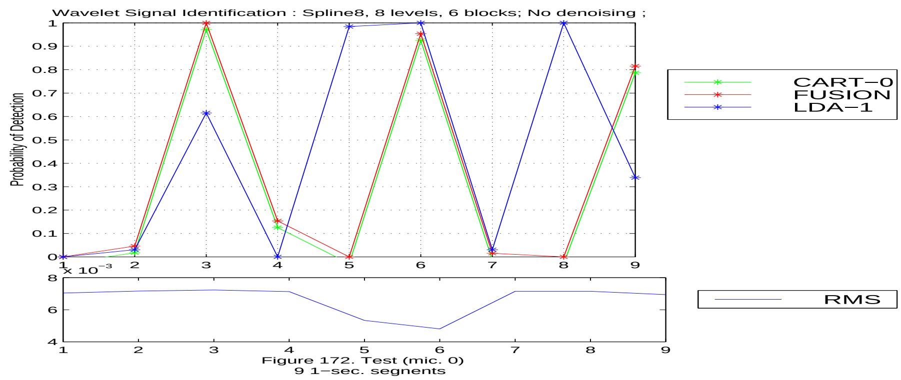

Figure 8. Results of testing of 9 O-signals which did not participate in the training stage.

5. CONCLUSIONS

We presented preliminary results of the application of the algorithm that were developed and implemented for acoustics differentiation. This was modified in this work to handle detection of symptoms of a certain disease in recordings of pulses in the radial artery of the human body. These two a problems have quasi-periodic structure in the underlying signals.

This medical detection problem is important because it enables to have express diagnostics for many diseases without the need to have an invasive procedure. In addition, this can be achieved in real-time. The obtained results are promising. They hint of the possibilities to develop robust diagnostic and prognostic systems on the basis of the proposed algorithm. These algorithms can be useful in other signal processing based diagnostics such as ECG, EEG, to name a few.

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