Convolution

Continuous

$$f * g(x) = \int f(y)g(x-y)dy$$

Discrete

$$f * g(x) = \frac{1}{N}\sum_{n=0}^{N-1} f(n)g(x-n)$$

Fourier

$$f(x,y) * g(x,y) \leftrightarrow F(u,v) \cdot G(u,v)$$

Correlation

Continuous

$$f \circ g(x) = \int f^*(y)g(x+y)dy = f^*(y) * g(-y)$$

Discrete

$$f \circ g(x) = \frac{1}{N}\sum_{n=0}^{N-1} f^*(n)g(x+n)$$

Fourier

$$f(x,y) \circ g(x,y) \leftrightarrow F^*(u,v) \cdot G(u,v)$$
$$f^*(x,y)g(x,y) \leftrightarrow F(u,v) \circ G(u,v)$$

Point Spread Function (PSF)

Response of a filter to a point image.
Let $P_{mn} = 1$ if $m = n = 0$ and 0 otherwise. Then

$$P'_{mn} = \sum_{i=-r}^{r}\sum_{j=-r}^{r} H_{ij}P_{m-i,n-j} = H_{mn}$$

Since convolution is linear when we know the effect of a filter on a point image we know it on any image.

Theorem: A linear shift-invariant operator must be a convolution operator in space.