Gray-value morphological processing

The techniques of morphological filtering can be extended to gray-level images. To simplify matters we will restrict our presentation to structuring elements, ***B***, that comprise a finite number of pixels and are convex and bounded. Now, however, the structuring element has gray values associated with every coordinate position as does the image ***A***.

* *Gray-level dilation*, $D_G(*)$, is given by:

Dilation - $$D_G(\mathbf{A},\mathbf{B}) = \max_{[j,k]\in \mathbf{B}} \{a[m-j,n-k]+b[j,k]\}$$

For a given output coordinate [*m*,*n*], the structuring element is summed with a shifted version of the image and the maximum encountered over all shifts within the *J* x *K domain* of ***B*** is used as the result. Should the shifting require values of the image ***A*** that are outside the *M* x *N* domain of ***A***, then a decision must be made as to which model for image extension, as described in Section 9.3.2, should be used.

* *Gray-level erosion*, $E_G(*)$, is given by:

Erosion - $$E_G(\mathbf{A},\mathbf{B}) = \min_{[j,k]\in \mathbf{B}} \{a[m+j,n+k]-b[j,k]\}$$

The duality between *gray-level erosion* and *gray-level dilation*--the gray-level counterpart of eq. --is somewhat more complex than in the binary case:

Duality - $$E_G(\mathbf{A},\mathbf{B}) = -D_G(-\tilde{\mathbf{A}},\mathbf{B})$$

where " $-\tilde{\mathbf{A}}$ " means that *a*[*j*,*k*] -> -*a*[-*j*,-*k*].

The definitions of higher order operations such as *gray-level opening* and *gray-level closing* are:

Opening - $$O_G(\mathbf{A},\mathbf{B}) = D_G(E_G(\mathbf{A},\mathbf{B}),\mathbf{B})$$

Closing - $$C_G(\mathbf{A},\mathbf{B}) = -O_G(-\mathbf{A},-\mathbf{B})$$

The important properties that were discussed earlier such as idempotence, translation invariance, increasing in A, and so forth are also applicable to gray level morphological processing. The details can be found in Giardina and Dougherty .

In many situations the seeming complexity of gray level morphological processing is significantly reduced through the use of symmetric structuring elements where *b*[*j*,*k*] = *b*[-*j*,-*k*]. The most common of these is based on the use of ***B*** = *constant* = 0. For this important case and using again the domain [*j*,*k*] $\subset$***B***, the definitions above reduce to:

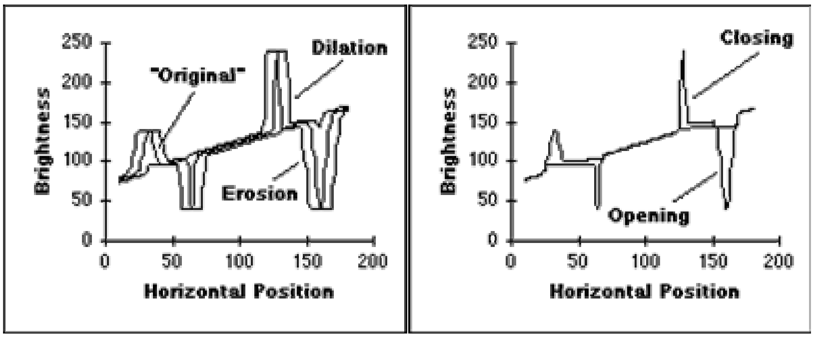
Dilation - $D_G(A,B) = \max_{[j,k]\in B}\{a[m-j,n-k]\} = \max_B(A)$

Erosion - $E_G(A,B) = \min_{[j,k]\in B}\{a[m-j,n-k]\} = \min_B(A)$

Opening - $O_G(A,B) = \max_B(\min_B(A))$

Closing - $C_G(A,B) = \min_B(\max_B(A))$

The remarkable conclusion is that the *maximum filter* and the *minimum filter*, introduced in Section 9.4.2, are gray-level dilation and gray-level erosion for the specific structuring element given by the shape of the filter window with the gray value "0" *inside* the window. Examples of these operations on a simple one-dimensional signal are shown in Figure 45.



a) Effect of 15 x 1 *dilation* and *erosion* **b)** Effect of 15 x 1 *opening* and *closing*

Figure 45: Morphological filtering of gray-level data.

For a rectangular window, *J* x *K*, the two-dimensional maximum or minimum filter is separable into two, one-dimensional windows. Further, a one-dimensional maximum or minimum filter can be written in incremental form. (See Section 9.3.2.) This means that gray-level dilations and erosions have a computational complexity per pixel that is *O*(*constant*), that is, independent of *J* and *K*. (See also Table 13.)

The operations defined above can be used to produce morphological algorithms for smoothing, gradient determination and a version of the Laplacian. All are constructed from the primitives for *gray-level dilation* and *gray-level erosion* and in all cases the *maximum* and *minimum* filters are taken over the domain $[j,k]\in B$.

Morphological smoothing

This algorithm is based on the observation that a *gray-level opening* smoothes a gray-value image from above the brightness surface given by the function *a*[*m*,*n*] and the *gray-level closing* smoothes from below. We use a structuring element ***B*** based on eqs. and .

$$\begin{aligned} MorphSmooth(A,B) &= C_G(O_G(A,B),B) \\ &= \min(\max(\max(\min(A)))) \end{aligned}$$

Note that we have suppressed the notation for the structuring element ***B*** under the *max* and *min* operations to keep the notation simple.

Morphological gradient

For linear filters the gradient filter yields a vector representation (eq. (103)) with a magnitude (eq. (104)) and direction (eq. (105)). The version presented here generates a morphological estimate of the *gradient magnitude*:

$$\begin{aligned} \mathit{Gradient}(\mathrm{A},\mathrm{B}) &= \frac{1}{2}\left(D_G(\mathrm{A},\mathrm{B}) - E_G(\mathrm{A},\mathrm{B})\right) \\ &= \frac{1}{2}\left(\max(\mathrm{A}) - \min(\mathrm{A})\right) \end{aligned}$$

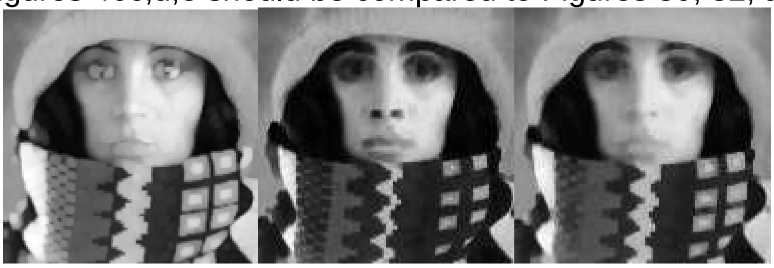
Morphological Laplacian

The morphologically-based Laplacian filter is defined by:

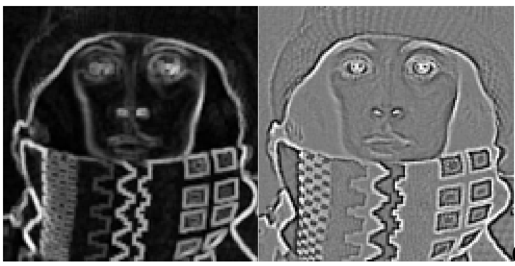
$$\begin{aligned} \mathit{Laplacian}(\mathrm{A},\mathrm{B}) &= \frac{1}{2}\left(\left(D_G(\mathrm{A},\mathrm{B}) - \mathrm{A}\right) - \left(\mathrm{A} - E_G(\mathrm{A},\mathrm{B})\right)\right) \\ &= \frac{1}{2}\left(D_G(\mathrm{A},\mathrm{B}) + E_G(\mathrm{A},\mathrm{B}) - 2\mathrm{A}\right) \\ &= \frac{1}{2}\left(\max(\mathrm{A}) + \min(\mathrm{A}) - 2\mathrm{A}\right) \end{aligned}$$

Summary of morphological filters

The effect of these filters is illustrated in Figure 46. All images were processed with a 3 x 3 structuring element as described in eqs. through . Figure 46e was contrast stretched for display purposes using eq. (78) and the parameters 1% and 99%. Figures 46c,d,e should be compared to Figures 30, 32, and 33.



a) Dilation **b)** Erosion **c)** Smoothing



d) Gradient **e)** Laplacian

Figure 46: Examples of gray-level morphological filters.