Double regularization approach to iterative blind multispectral image restoration

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ABSTRACT

In this paper, we present a new iterative blind multispectral image restoration algorithm based on double regularization (DR). The motivation for DR when applied to multispectral restoration lies in its effectiveness towards edge preservation in joint blur identification and image restoration. With consideration for both the intra- and inter-channel blurring function in the multiple-input multiple-output (MIMO) systems, an alternating minimization (AM) procedure with conjugate gradient optimization (CGO) scheme is formulated to implement restoration iteratively. The derivation of DR optimization shows that optimal restoration result can be achieved even when the MIMO systems suffer from inter-channel interference. Experimental results show that it is effective in performing blind multichannel restoration when applied to color images.

Keywords: blind image deconvolution, multispectral restoration, double regularization, multiple-input multiple-output

1. INTRODUCTION

Blind image restoration deals with the estimation of the original image from the degraded image using the partial information about the imaging system. It is an ill-posed problem as the uniqueness and stability of the solutions is not guaranteed [1]. In many applications such as super-resolution and microscopy imaging, multiple degraded images of a single scene become available while the original image and the blurs remain unknown. There are some rather successful works on development of multispectral restoration, which exploit the benefits of single-channel restoration techniques [2],[10],[11].

In [3], a greatest common divisor (GCD) approach in z-domain is proposed for multispectral restoration. However, exact GCD is not desirable because the quantization error or additive noise can lead to trivial solution. Using the nullspace concept, eigenstructure-based multispectral restoration algorithm can perform direct restoration in the noise-free environment [4]. Subspace and likelihood-based techniques have also been developed in [5],[6]. The algorithms work by first estimating the blurring function using min-eigenvector, followed by image restoration based on these estimates. However, the results are sensitive towards noise.

Another research direction involves extending the regularization theory to address blind image deconvolution. Double regularization (DR) decomposes the blind single-channel problem into two symmetric processes of blur identification and image restoration, which are minimized by alternating minimization (AM) [7]-[9]. Total variation (TV) has also been incorporated into DR to achieve edge preservation and noise suppression [8]. A promising attempt has been made by utilizing the multispectral eigenvector-based method (EVAM) as a regularization term in the framework of TV [10]. A direct extension of DR has also been applied successfully in color image restoration [11].

In view of this, we derive a new algorithm for iterative blind multispectral image restoration based on DR theory. With consideration for both the intra- and inter-channel blurring in the MIMO systems, an AM with conjugate gradient optimization (CGO) scheme is formulated to implement restoration iteratively. Although the algorithm is conceptually similar to those developed for single-channel problem, there are some important theoretical and practical differences when applied to the MIMO systems such as inter-channel blurring interference.

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2. PROBLEM FORMULATION

Let K be the number of FIR channels. Consider a MIMO linear degradation system, the ith-channel degradation process can be modeled as [2]:

\[ g_i = \sum_{j=1}^{K} H_{ij} f_j + n_i, \quad i = 1, 2, \cdots K \]  

where \( f_i \), \( g_i \), and \( n_i \) represent the lexicographically ordered original image, observed degraded image, and additive white Gaussian noise (AWGN) in the ith-channel, respectively. \( H_{ij} \) and \( H_{ij} (i \neq j) \) are the block-circulant matrices that denote the intra-channel and inter-channel blurring functions, respectively.

To tackle the ill-posed nature of image restoration, regularization theory has been employed in our scheme as it is effective towards edge preservation and noise suppression. Tikhonov-Miller regularization defines the criterion to determine the feasible restoration solutions. In terms of the ith-channel, it can be as described by:

\[ \| g_i - \sum_{j=1}^{K} H_{ij} \hat{f}_j \| \leq \| n_i \| \leq \epsilon_i^2 \]  

and

\[ \| C_i \hat{f}_i \| \leq E_i^2 \]  

where \( \| . \| \) represents the \( L_2 \)-norm, \( \hat{f}_i \) is the estimated image, \( \epsilon_i \) and \( E_i \) are the upper bounds related to the noise and the estimated image, respectively. \( C_i \) is the regularization operator and usually a high-pass filter. The term in (2) is the data-fidelity term, while the term in (3) is the regularization term to suppress the noise. In order to be able to perform joint blur identification and image restoration, double regularization is extended by adding on additional regularization term in the blur-domain [7],[8]:

\[ \sum_{j=1}^{K} \| D_j \hat{h}_j \| \leq \delta_i^2 \]  

where \( D_j \) is the regularization operator associate with the blurring function \( \hat{h}_j \), which is similar to \( C_i \). This term imposes the smoothness constraint \( \delta_i^2 \) on the estimated blurring functions.

For the ith-channel, the double quadrature formula is formulated as:

\[ J_i(\hat{f}_i, \hat{h}_i | g_i) = \frac{1}{2} \| g_i - \sum_{j=1}^{K} H_{ij} \hat{f}_j \|^2 + \frac{\alpha_i}{2} \| C_i \hat{f}_i \|^2 + \frac{\beta_i}{2} \sum_{j=1}^{K} \| D_j \hat{h}_j \|^2 \leq \frac{3}{2} \epsilon_i^2 \]  

where \( \alpha_i = (\epsilon_i / E_i)^2 \), \( \beta_i = (\epsilon_i / \delta_i)^2 \).

Thus, we have the cost function for the blind K-channel image restoration:

\[ J(\hat{f}, \hat{h} | g) = \sum_{i=1}^{K} J_i(\hat{f}_i, \hat{h}_i | g_i) \]  

where \( \hat{f} = \{ \hat{f}_1, \cdots, \hat{f}_K \}, \hat{g} = \{ \hat{g}_1, \cdots, \hat{g}_K \}, \hat{h} = \{ \hat{h}_1, \cdots, \hat{h}_K \} \).

The cost function in (6) consists of image- and blur-domain terms. As it is computationally intensive to perform joint optimization, we project the overall cost function \( J(\hat{f}, \hat{h} | g) \) into the image-domain cost function \( J(\hat{f} | g) \), and blur-domain cost function \( J(\hat{h} | g) \). An optimization procedure called AM is then developed to minimize them iteratively. The AM procedure can be summarized as:

(i) Initialize \( \hat{h} \), \( \hat{f} \)

(ii) Minimize the sub-domain cost functions iteratively:
\[ J(\hat{f} \mid g) = \frac{1}{2} \sum_{m=1}^{K} \| g_m - \sum_{j=1}^{K} H_j \hat{f}_j \|^2 + \alpha \| C \hat{f} \|^2 \]  
(7)

\[ J(\hat{h} \mid g) = \frac{1}{2} \sum_{m=1}^{K} \| g_m - \sum_{j=1}^{K} H_j \hat{f}_j \|^2 + \beta \sum_{j=1}^{K} \| D_j \hat{h}_j \|^2 \]  
(8)

(iii) Stop when termination condition is satisfied.

The AM procedure enjoys algorithmic simplicity as both \( J(\hat{f} \mid g) \) and \( J(\hat{h} \mid g) \) are convex, hence guaranteeing their convergence in each domain.

3. SUB-DOMAIN MINIMIZATION

Conjugate gradient optimization (CGO) is used in conjunction with AM to optimize the image- and blur-domain cost functions. CGO utilizes conjugate direction instead of local gradient to search for the minima. Therefore, it can achieve faster convergence when compared with steepest descent method. It also requires less storage requirement and computational complexity when compared with Quasi-Newton method. CGO is ideal in this application as the given Hessian matrices are sparse, leading to fast convergence in small number of iterations.

For mathematical simplicity, we omit the “\( ^{\hat{\ }} \)” in the following subsections, which denote the estimated value.

3.1 Minimization of image-domain cost function

Rewriting the image-domain cost function (7) in terms of \( f \):

\[ J(\hat{f} \mid g) = \frac{\alpha}{2} \| g - Hf \|^2 + \frac{1}{2} \| Cf \|^2 \]  
(9)

where

\[ f = \begin{bmatrix} f_1 \\ \vdots \\ f_K \end{bmatrix}, \quad g = \begin{bmatrix} g_1 \\ \vdots \\ g_K \end{bmatrix}, \quad H = \begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1K} \\ H_{21} & H_{22} & \cdots & H_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ H_{K1} & H_{K2} & \cdots & H_{KK} \end{bmatrix}, \quad C = \begin{bmatrix} \sqrt{\alpha_1} C_1 & 0 & \cdots & 0 \\ 0 & \sqrt{\alpha_2} C_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{\alpha_K} C_K \end{bmatrix} \]

The gradient of \( J(\hat{f} \mid g) \) with respect to \( f \) is given by:

\[ \nabla J_f = \frac{\partial J(\hat{f} \mid g)}{\partial f} = -H^T(g - Hf) + C^T Cf \]

(10)

The mathematical formulations of image restoration based on CGO are developed as follows:

(i) Initialize the conjugate vector \( q \):

\[ q^{(0)} = -\nabla J_f^{(0)} \]  
(11)

(ii) Update the \((k+1)\)-th iteration image estimate:

\[ f^{(k+1)} = f^{(k)} + \eta^{(k)} q^{(k)} \]  
(12)

where

\[ \eta^{(k)} = \frac{\| \nabla J_f^{(k)} \|^2}{\| Hq^{(k)} \|^2 + \| Cq^{(k)} \|^2} \]

(13)

(iii) Update the \((k+1)\)-th iteration conjugate vector:
\[ q^{(k+1)} = -\nabla J_f^{(k)} + \rho^{(k)} q^{(k)} \]  

where

\[ \rho^{(k)} = \frac{\| \nabla J_f^{(k+1)} \|^2}{\| \nabla J_f^{(k)} \|^2} \]

(iv) Repeat steps (ii) to (iii) until convergence or a maximum number of iterations is reached.

3.2 Minimization of blur-domain cost function

Let us consider the simple degradation model in the \(i\)th-channel with inter- and intra-channel blurs.

\[ g_i(x, y) = \sum_{j=1}^{K} \sum_{m, n \in S_h} h_{ij}(m, n) f_j(x-m, y-n) \]  

(16)

The 2-D convolution in (16) can be written in matrix-vector representation as:

\[ F_i = \sum_{j=1}^{K} F_j \text{vec}(h_j) \]  

(17)

where \( S_h \) denotes the support size of \( h_j \), which we assume to be \( P \times Q \). \( \text{vec}(h_j) \) is the \( PQ \times 1 \) vector by concatenating column of \( h_j \). \( F_j \) is an array of shift-images corresponding to the entries of \( \text{vec}(h_j) \).

\[ F_j = \begin{bmatrix} f_j(-\frac{P-1}{2}, \frac{Q-1}{2}) & f_j(-\frac{P-1}{2}, \frac{Q-1}{2}) & \cdots & f_j(-\frac{P-1}{2}, \frac{Q-1}{2}) \\ f_j(-\frac{P-1}{2}, -\frac{Q-1}{2}) & f_j(-\frac{P-1}{2}, \frac{Q-1}{2}) & \cdots & f_j(-\frac{P-1}{2}, \frac{Q-1}{2}) \\ \vdots & \vdots & \ddots & \vdots \\ f_j(-\frac{P-1}{2}, -\frac{Q-1}{2}) & f_j(-\frac{P-1}{2}, -\frac{Q-1}{2}) & \cdots & f_j(-\frac{P-1}{2}, -\frac{Q-1}{2}) \end{bmatrix} \]  

(18)

where shift-image \( f_j^{(-x,-y)} \) means shifting the image with shiftsize \((x, y)\) in the vertical and horizontal directions, respectively.

Thus, rewriting the blur-domain cost function (8) as the formation of (17), we have

\[ J(h | g) = \| g - Fh \|^2 + \| Dh \|^2 \]  

(19)

where

\[ h = \begin{bmatrix} \text{vec}(h_1) \\ \text{vec}(h_2) \\ \vdots \\ \text{vec}(h_K) \end{bmatrix}, g = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_K \end{bmatrix}, F_i = [F_{i1}, F_{i2}, \ldots, F_{iK}] \]

\[ F = \begin{bmatrix} F_1 & 0 & \cdots & 0 \\ 0 & F_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & F_K \end{bmatrix}, D = \begin{bmatrix} D_{i1} & 0 & \cdots & 0 \\ 0 & D_{i2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D_{iK} \end{bmatrix} \]

The formulation involved in the optimization of \( J(h | g) \) is quite similar to image-domain optimization, which is based on CGO.

4. RESULTS

We have developed the theory in pervious section for complete multi-channel restoration, in which the inter- and intra-channel blurring function are arbitrary.

In the following, we will look at a specific case of MIMO systems, namely color images to test the performance of the propose method. In this experiment, we assume that the inter-channel blurring inference to be small. A RGB-channel
“Lena” image of size 256 \times 256 in Fig. 1(a) is chosen as the test image. Each channel is passed through different Gaussian blurring functions (5 \times 5, \sigma_{red} = 2.4, \sigma_{green} = 2.6, \sigma_{blue} = 2.8). Further, AWGN is added to each channel (30, 35, 40 dB SNR in R, G, B channels) to produce the degraded image in Fig. 1(b), respectively. For the performance evaluation, we use the percentage mean square error as the objective measure [10]:

\[
PMSE(f) = \frac{\|\hat{f} - f\|}{\|f\|}
\] (20)

The final restored result is given in Fig. 1(c). It is observed that most visual clarity has been recovered, especially near the feather and the hat regions. This is confirmed by a \(PMSE(\hat{f})\) of 0.0063. The good result obtained by our technique illustrates its advantages.

![Fig. 1. Blind multispectral image restoration (a) Original color image. (b) Degraded color image. (c) Restored color image.](image)

5. CONCLUSIONS

We have proposed an iterative blind multispectral image restoration algorithm based on double regularization theory. The proposed method makes use of alternating minimization (AM) with conjugate gradient optimization (CGO) to minimize a proposed multispectral cost function iteratively. With the consideration for both the intra- and inter-channel blurring in the MIMO system, our approach can apply to multispectral restoration problem efficiently. Experimental results show that it is effective in performing blind multispectral restoration when applied to color images.

REFERENCE