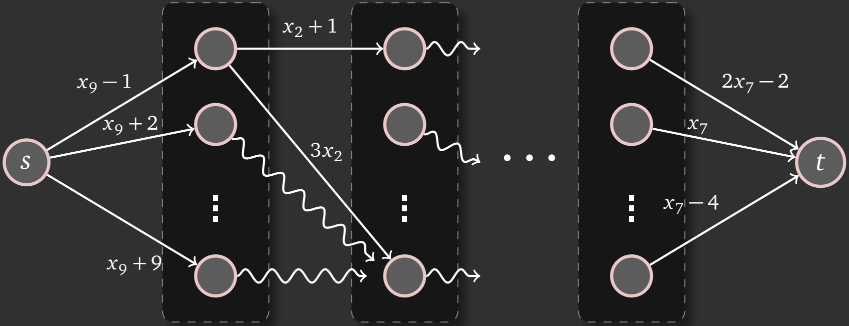
IDENTITY TESTING & LOWER BOUNDS
FOR
READ-k OBLIVIOUS ABPS

Ben Lee Volk

Joint with

Matthew Anderson
Michael A. Forbes
Ramprasad Saptharishi
Amir Shpilka

READ-ONCE OBLIVIOUS ABPS



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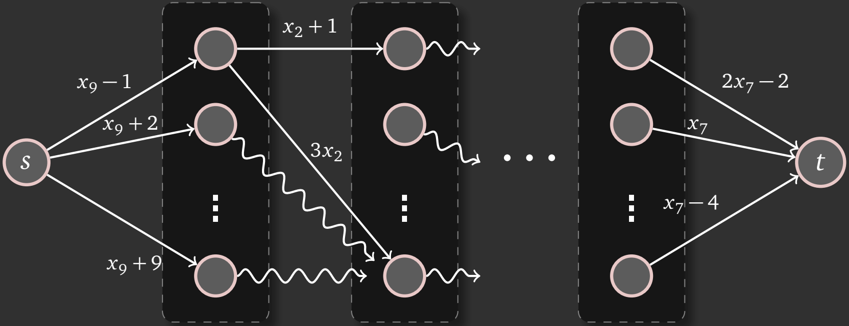

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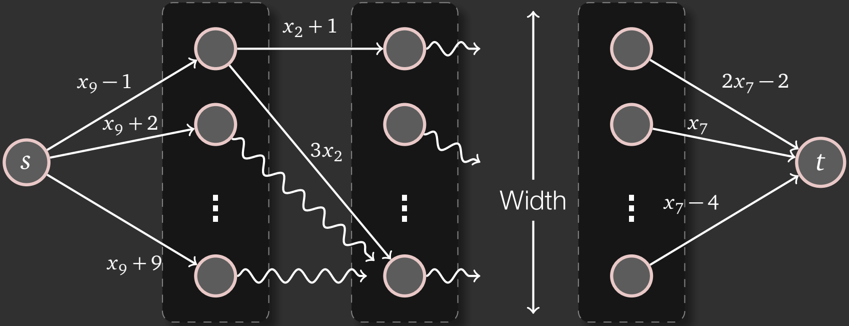

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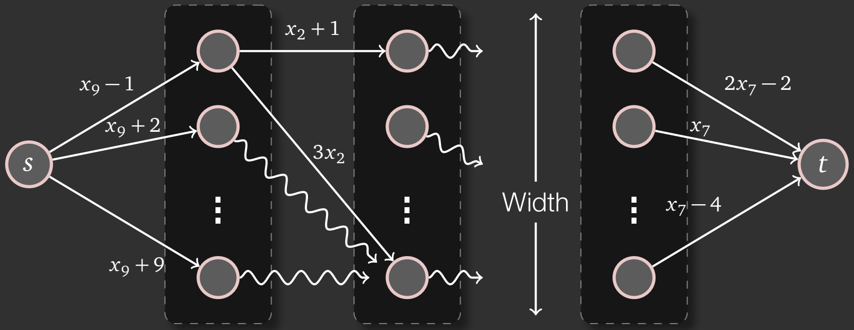

Equivalently: f is the $(1, 1)$ entry of the iterated matrix product

$$\prod_{i=1}^{n} M_i(x_{\pi(i)})$$

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(def: same as before except that now every variable appears in at most k layers)

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- $\exp(n/2^k)$ lower bounds **[Okolnishnikova, Borodin-Razborov-Smolensky]** even for randomized and non-deterministic variants
- PRG with seed length $\sqrt{s}$ for size-s programs **[Impagliazzo-Meka-Zuckerman]**

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Lower Bound: There is a polynomial $f \in \mathrm{VP}$ that requires read-k oblivious ABPs of width $\exp(n/k^k)$.

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PIT: There is a white-box* PIT algorithm for read-k oblivious ABPs, of running time $\exp(n^{1-1/2^{k-1}})$.

*only the order in which the variables appear is important

EVALUATION DIMENSION

Reminder: $f \in \mathbb{F}[x_1, \ldots, x_n]$, $S \subseteq [n]$.

$$\text{eval-dim}_{S,\overline{S}}(f) = \dim\ \text{span}\ \left\{ f|_{\mathbf{x}_S = \alpha} \mid \alpha \in \mathbb{F}^{|S|} \right\}.$$

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Characterizes ROABP complexity:

Theorem [Nisan]: f has ROABP of width w in variable order $x_1, x_2, \ldots, x_n$ iff for every $i \in [n]$,

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(same as rank of partial derivative matrix)

WARM-UP: 2-PASS ABP

Same as ROABP but with two “passes”:

x_1	x_2	$\cdots$	x_{n-1}	x_n	x_1	x_2	$\cdots$	x_{n-1}	x_n

$$f = \left(M_1^1(x_1)M_2^1(x_2)\cdots M_n^1(x_n)\cdot M_1^2(x_1)M_2^2(x_2)\cdots M_n^2(x_n)\right)_{(1,1)}$$

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Fixing $x_1 = \alpha_1, x_2 = \alpha_2$:

$$\left(N^1(\alpha_1,\alpha_2)M_3^1(x_3)\cdots M_n^1(x_n)\cdot N^2(\alpha_1,\alpha_2)M_3^2(x_3)\cdots M_n^2(x_n)\right)_{(1,1)}$$

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So eval-dim$_{[i],\overline{[i]}}(f) \le w^4$. $\implies$ f has width w^4 ROABP.

GENERALIZE: k-PASS ABP

Theorem: If f is computed by a width-w k-pass ABP in variable order $x_1, x_2, \ldots, x_n$, then for every $i \in [n]$, eval-dim$_{[i],\overline{[i]}}(f) \leq w^{2k}$.

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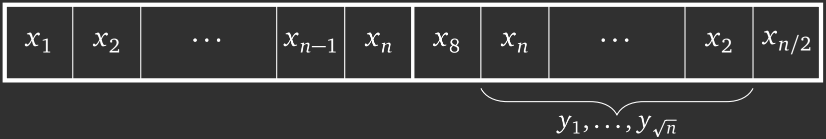

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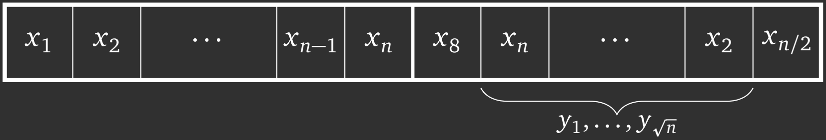

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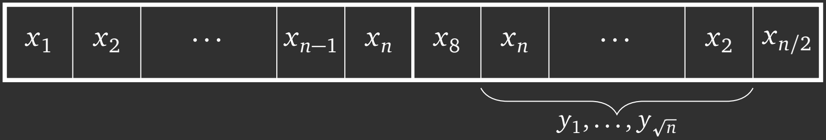

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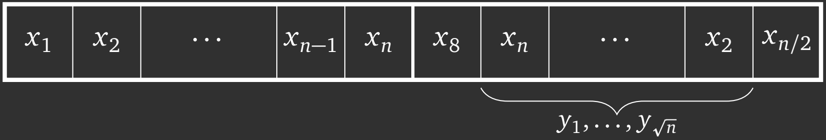

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What you get is a 2-pass ABP over $\mathbf{y}$ vars. In other words, ignoring $\overline{\mathbf{y}}$, for every $i \in [\sqrt{n}]$, eval-dim$_{[i],\overline{[i]}}(f) \leq w^4$.

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Naturally generalizes to k passes with different orders.

PIT FOR k-PASS, DIFFERENT ORDERS

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This is still not a general read-k oblivious ABP!

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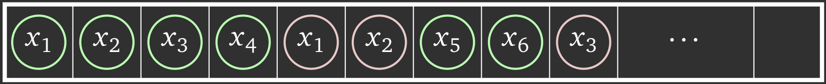

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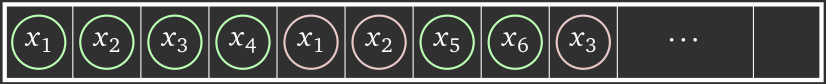

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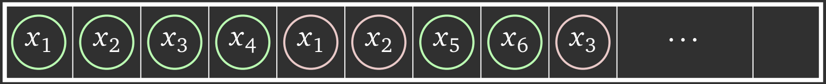

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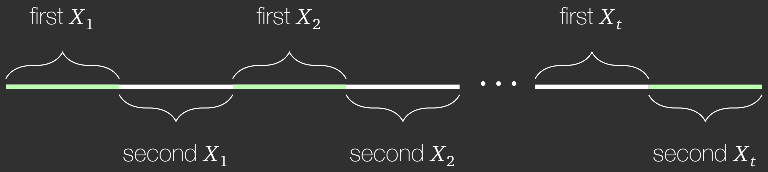

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This structure is enough to carry out the original argument: with respect to the variables $\mathbf{y}$ in the regularly interleaving sequence ($|\mathbf{y}| \approx \sqrt{n}$), the evaluation dimension is at most w^4.

Generalizes to read-k: apply Erdős-Szekeres to every sequence and make every pair regularly-interleaving.

Wrap-up: PIT algorithm with running time $\exp(n^{1-1/2^{k-1}})$ for read-k oblivious ABPs.

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- This is very close to being true

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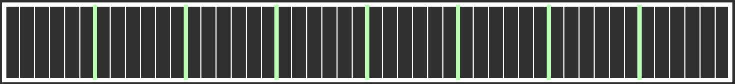

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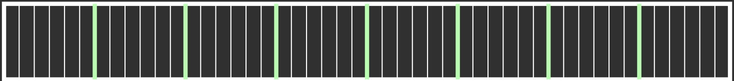

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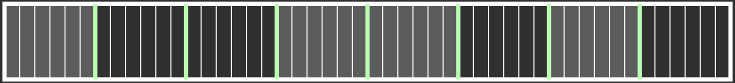

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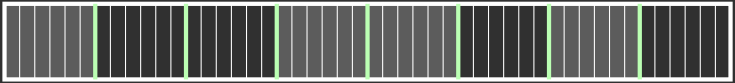

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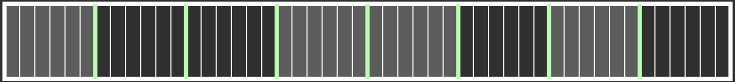

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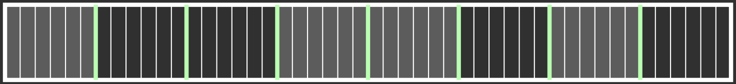

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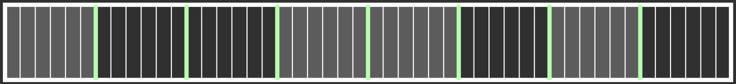

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what's left is to find a polynomial such that eval-dim$_{S,T} \geq 2^{\min\{|S|,|T|\}}$

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PIT: A white-box PIT algorithm for read-k oblivious ABPs, with running time $\exp(n^{1-1/2^{k-1}})$.

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THANK YOU