Sub-linear Algorithms	November 30, 2015

Homework 3

Lecturer: Ronitt Rubinfeld	*Due Date: December 14, 2015*

1. (Property testing of the clusterability of a set of points.) Given a set X of points in any metric space. Assume that one can compute the distance between any pair of points in one step. Say that X is (k, b)-diameter clusterable if X can be partitioned into k subsets (clusters) such that the maximum distance between any pair of points in a cluster is b. Say that X is ϵ-far from (k, b)-diameter clusterable if at least $\epsilon|X|$ points must be deleted from X in order to make it (k, b)-diameter clusterable.

   Show how to distinguish the case when X is (k, b)-diameter clusterable from the case when X is ϵ-far from $(k, 2b)$-diameter clusterable. Your algorithm should use polynomial in $k, 1/\epsilon$ queries. It is possible to get an algorithm which uses $O((k^2 \log k)/\epsilon)$ queries.

2. Assume that your computational model is such that a query returns a single bit. In such a model, show that any algorithm making q queries can be made into a *nonadaptive* (i.e., where the queries do not depend on the results of any previous queries) tester that uses only 2^q queries.

3. (Canonical forms for graph property testers for the adjacency matrix model). Define a graph property to be a property that is preserved under graph isomorphism – i.e., if G has the property and G' is isomorphic to G, then G' must also have the property. Show that any adaptive algorithm for property testing which makes q queries, can be made nonadaptive algorithm using only $O(q^2)$ queries.