Probabilistic Data Structures

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Membership testing

Hash table

An array of m elements and a hash function h

- How do we keep track of collisions?
  - How expensive is it?
  - What if we don't keep track?

Bloom filter

Use k hash functions $h_1, h_2, \ldots, h_k$ on a *bit array*

- No false negatives
- Saves space
- Constant time to add an element

Bloom filter – false positives

After n insertions,

$$Pr(bit = 0) = \left(1 - \frac{1}{m}\right)^{kn}$$

Probability of false positive:

$$\left(1 - \left(1 - \frac{1}{m}\right)^{kn}\right)^{k} \approx \left(1 - e^{-\frac{kn}{m}}\right)^{k}$$

Use in streaming scenarios

Scalable Bloom filter

- Add an arbitrary number of elements
- Constant bound on false positives
- Becomes expensive in terms of space

Stable Bloom filter

Goals:

- Use constant memory
- Evict stale data

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Results:

- The number of 0s in the array converges
- We can use this to limit false positives
- False negatives are introduced

How can we save more information?

Multisets – stream summary

We'd like to get a histogram of the elements in the stream

- Point queries

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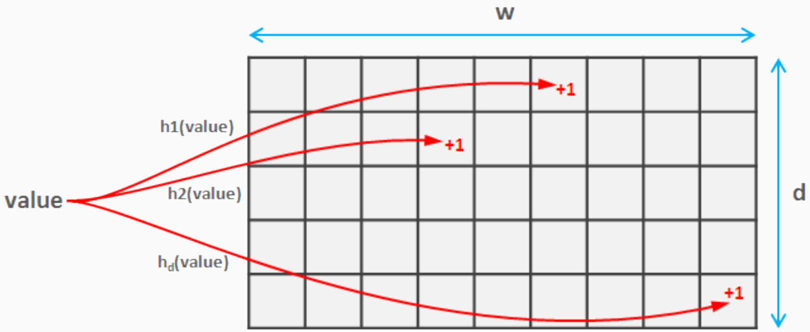
- "Heavy hitters"
- Quantile queries

Count-min sketch

- Split each of *k* hash functions of bloom filter into separate array of size *m*
- Use counters
- We gain the ability to delete



Questions?