COMPLEXITY: Exercise No. 5
due Jun 2

1. Are the following problems **NP**-complete or polynomial? (prove)

   DOMINATING PATH:
   INSTANCE: A graph $G = (V, E)$, positive integer K.
   QUESTION: Does G contain a simple path of length at most K whose vertices form a dominating set?

   CONNECTED DOMINATING SET:
   INSTANCE: A graph $G = (V, E)$, positive integer K.
   QUESTION: Does G contain a dominating set S with at most K vertices such that the subgraph of G induced by S (i.e., the graph $G_S = (S, E \cap S \times S)$) is connected ?

   MINIMUM LEAF SPANNING TREE:
   INSTANCE: A graph $G = (V, E)$, a positive integer K.
   QUESTION: Is there a spanning tree for G such that the number of leaves in the tree is at most K ?

   MAXIMUM LEAF SPANNING TREE:
   INSTANCE: A graph G, a positive integer K.
   QUESTION: Is there a spanning tree for G such that the number of leaves in the tree is at least K ?

   BIPARTITE MAX CUT:
   INSTANCE: A bipartite graph $G = (V, E)$, a positive integer K.
   QUESTION: Is there a partition of V into disjoint sets V_1 and V_2 such that the number of edges from E that have one endpoint in V_1 and one endpoint in V_2 is at least K ?

   FLOW WITH MULTIPLIERS: (Hint: use SUBSET SUM)
   INSTANCE: A directed graph $G = (V, E)$, vertices $s, t \in V$, positive integer multiplier $h(v)$ for every vertex $v \in V - \{s, t\}$, positive integer capacity $c(e)$ for every $e \in E$, and a positive integer K.
   QUESTION: Is there a *non-negative integer* function f such that:

   (a) $f(e) \leq c(e)$ for all $e \in E$.

   (b) for each $v \in V - \{s, t\}$, $\sum_{u:(u,v)\in E} h(v)f((u, v)) = \sum_{u:(v,u)\in E} f((v, u))$.

   (c) $\sum_{u:(u,t)\in E} f((u, t)) \geq K$.

2. a. Show that INDEPENDENT SET remains **NP**-complete even if the input graph has no clique of size 3.

   b. Show that INDEPENDENT SET remains **NP**-complete even if the input graph has maximum degree ≤ 3.