COMPLEXITY: Exercise No. 3
due May 5

1. Give a NDTM that accepts the *complement* of L_{pal} with space $O(\log n)$ and time $O(n)$

2. Show that a language decided by a k-tape NDTM operating within time $f(n)$ can be decided by a 2-tape NDTM in time $f(n)$ (Assume that $f(n)$ is a proper complexity function, and that $f(n) = \omega(n)$).

3. a. For a language L define $L^* = \{x_1 x_2 \cdots x_k : k \geq 0, x_1, \ldots, x_k \in L\}$. Show that if $L \in \mathbf{NP}$ then $L^* \in \mathbf{NP}$.
   b. Show that if $L \in \mathbf{P}$ then $L^* \in \mathbf{P}$.

4. a. For a language L and a function $f : N \to N$ satisfying $f(n) \geq n$, define $\mathrm{Pad}(L, f(n)) = \{x\#^{f(|x|)-|x|} : x \in L\}$ where $\#$ is a symbol not in the alphabet of L. A complexity class $\mathbf{C}$ has the *padding property* if for any integer $k \geq 1$, $L \in \mathbf{C} \iff \mathrm{Pad}(L, n^k) \in \mathbf{C}$. Which of the following classes have the padding property: $\mathbf{P}, \mathbf{NP}, \mathbf{SPACE}(n)$? **Remark:** Until it will be proved in class, you may assume $\mathbf{PSPACE} \neq \mathbf{SPACE}(n)$.
   b. Prove $\mathbf{NP} \neq \mathbf{SPACE}(n)$ and $\mathbf{P} \neq \mathbf{SPACE}(n)$.

5. a. Show that if f, g are proper complexity function then so are $f + g, f \cdot g, 2^g$ and $f(g)$ (for the last part, assume $f(n) \geq n$).
   b. Show that $\lceil \log n \rceil, n!$ and $\lceil n^{1/3} \rceil$ are proper complexity functions.
   c. Show that $\log \log n$ is *not* a proper complexity function.