COMPLEXITY: Exercise No. 1
due April 23

1. Prove or disprove:

   (a) $(2n)! = O(n!^2)$.

   (b) $\log(n!) = \Theta(n \log n)$.

   (c) If $f = o(g)$ then there is h such that $f = o(h)$ and $h = o(g)$.

   (d) There is a function f such that $f(n) = O(n^{1+\epsilon})$ for every $\epsilon > 0$ but $f(n) = \omega(n)$.

   (e) If f is monotone positive function, $f(n) = O(n)$ and $f(n) \neq o(n)$ then $f(n) = \Theta(n)$.

2. a. Describe explicitly a DTM that counts from k to 0 using $O(\log k)$ space. What is the running time?

   b. Sketch a DTM that accepts L_{pal} (the language of palindromes) with space $O(\log n)$ and time $O(n^2/\log n)$.

3. Describe explicitly an $O(n^2)$-time 2-tape DTM that for any input of length n, it writes 1^{n^2} (i.e., a string of length n^2 containing only the letter 1) on the second tape.

4. Let M be a k-tape DTM with input and output which always stops with time complexity $t(n)$ and space complexity $s(n)$. Prove that there is a constant c (which depends on M) such that $t(n) \leq nc^{s(n)}$ (assume $s(n) > 0$ for all n).