# Final Exam - Program Verification

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Please either print or write VERY CLEARLY. Submit a pdf file by e-mail or a hard copy by July 5. (A handwritten version can be scanned). You can use any literature. However, I would like to emphasize that the exam is an **INDIVIDUAL TASK and is NOT to be done in groups!!!**. Please submit together with the exam the signed declaration below.

**Declaration**

Student's Name:
Student's ID Number:
I certify hereby that:
1. This exam was prepared by me solely, without any assistance from others.
2. I have not assisted any other person in the preparation of his/her exam.
SIGNATURE:

**Grading:** Exercises 1-5 are 20 points each.

### Question 1

Show that the following problem is in PSPACE.
**Input:** a Buchi automaton $A$ and a TL(U) formula $\varphi$.
**Question:** Is there an $\omega$-string $u$ such that $u$ is accepted by $A$ and $u, 0 \models \varphi$?

### Question 2

Let $Z \times 2$ be a linear order defined as follows:

(a) Domain: The set of pairs $(i, a)$ where $i$ is an integer and $a \in \{0, 1\}$.

(b) The interpretation of $<$: $(i, a) < (j, b)$ iff either ($a = 0$ and $b = 1$) or ($a = b$ and $i < j$).

Show that there is an algorithm that for every MLO sentence $\varphi$ decides whether $\varphi$ is satisfiable in $Z \times 2$.

### Question 3

Two $\omega$-strings $u$ and $v$ over a finite alphabet $\Sigma$ are said to be $\sim_s$-equivalent if $u = {a_1}^{n_1}{a_2}^{n_2}\dots{a_k}^{n_k}\dots$ and $v = {a_1}^{m_1}{a_2}^{m_2}\dots{a_k}^{m_k}\dots$ where $a_k \in \Sigma$ and $0 < min(m_k, n_k)$ for every $k$. Prove that if $L$ is an $\omega$-regular language, then $L_1 = \{u \mid$ there is $v \in L$ such that $v \sim_s u$ and $v = a_1a_2\dots a_k\dots$ and $\forall i(a_i \neq a_{i+1})$ $\}$ is also $\omega$-regular.

### Question 4

Let $X$ be a set of finite strings over $\{0, 1\}^2$. Assume that the empty string is not in $X$.
$G(X)$ is a two person game defined as follows.
**Round $i$:** Player 1 chooses $a_i \in \{0, 1\}$. Player 2 replies by $b_i \in \{0, 1\}$.
**Winning conditions for $G(X)$:** Player 1 wins a play $\rho = (a_1, b_1)(a_2, b_2)\dots(a_k, b_k)\dots$ if there is $i$ such that $(a_1, b_1)(a_2, b_2)\cdots(a_i, b_i) \in X$. Otherwise Player 2 wins $\rho$.

The game $G_\omega(X)$ is defined like $G(X)$, but it has the following winning conditions.
**Winning conditions for $G_\omega(X)$:** Player 1 wins a play $\rho$ if $\rho \in X^\omega$; otherwise Player 2 wins $\rho$.
Prove

(a) Player 2 has a winning strategy in $G(X)$ iff he has a winning strategy in $G_\omega(X)$.

(b) Player 1 has a winning strategy in $G(X)$ iff he has a finite memory winning strategy in $G(X)$.

A strategy $\sigma$ for player 1 is called a finite memory iff there is a deterministic finite state automaton $A$ over alphabet $\{0, 1\}$ such that $\sigma((a_1, b_1)(a_2, b_2)\cdots(a_i, b_i)) = 1$ iff $b_1\dots b_i$ is accepted by $A$.

### Question 5

Let $\Sigma$ be a finite alphabet, $a_i \in \Sigma$ and $\Sigma_i \subseteq \Sigma$ for $i = 1, \dots, n$. Write $TL(U)$ fotmula $\varphi$ such that $u, 0 \models \varphi$ iff $u \in \Sigma_1^* a_1 \Sigma_2^* a_2 \cdots \Sigma_n^* a_n \Sigma^\omega$.

GOOD LUCK