Final Exam - Program Verification

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April 17, 2008

Please either print or write VERY CLEARLY. Submit a pdf file by e-mail on April 20. (A handwritten version can be scanned). You can use any literature. However I would like to emphasize that the exam should be made **INDIVIDUALLY and NOT in groups!!!**

Grading: Exercises 1-5 are 20 point each.

Exercise 1 $G = (V_1, V_2, E)$ *is a simple graph game of size* n *if*

1. V_i *are positions of player* i. *They have satisfy* $V_1 \cap V_2 = \emptyset$, $V_1 \cup V_2 \subseteq \{1, 2. \ldots n\}$. $V := V_1 \cup V_2$ *is the set of all positions.*

2. $E \subseteq V \times V$ *is the set of all possible moves.*

3. *In graph-theoretic terms,* V *is the set of nodes, and* E *the set of edges of graph* G. *They have to satisfy in addition that at least one edge is leaving each node.*

Winning conditions: *The winner of a play* $v_1 \to v_2 \to \ldots v_i \to v_{i+1} \to \cdots$ *is the player owning the least node which is visited infinitely often in the play.*

Show

1. *In a simple graph game one of the player has a memoryless winning strategy.*

2. $\sigma : V_1 \to V$ *is a memoryless strategy of the first player iff in the following graph* G_σ *the least node in every cycle is owned by the first player.*

The nodes of $G_\sigma := V_1 \cup V_2$. *The edges of* G_σ*:*

- *for $v \in V_2$, there is an edge from v to u in G_σ iff there is an edge from v to u in G.*

- *for $v \in V_1$. there is an edge from v to u iff $u = \sigma(v)$.*

Exercise 2 *Show that there is a weak Muller game such that for each node v in the first player's winning region, Player I has a memoryless strategy σ_v which is winning for the first player for the plays which starts from v , however, Player I has no uniform memoryless winning strategy. (A strategy is uniform winning strategy for Player I if it is a winning strategy for every node of his winning region.)*

Exercise 3 *For a string (or ω-string) s and an alphabet Σ the string $s\lfloor\Sigma$ is obtained from s by deleting all letters not in Σ. Let L_1 and L_2 be ω languages over alphabets Σ_1 and Σ_2. The ω-language $L_1|L_2$ is defined as follows: $s \in L_1|L_2$ iff $s\lfloor\Sigma_1 \in L_1$ and $s\lfloor\Sigma_2 \in L_2$. Show that if L_1 and L_2 are ω-regular languages then $L_1|L_2$ is ω-regular.*

Exercise 4 *For a natural number k, let $\omega \times k$ be a linear order defined as follows:*

1. *Domain: The set of pairs (i, n) where $i < k$ and n is a natural number.*

2. *The interpretation of <: $(i, n) < (j.m)$ iff either $i < j$ or ($i = j$ and $n < m$).*

Show that for every k there is an algorithm that for every MLO sentence φ decides whether φ is satisfiable in $\omega \times k$.
Hint: *You can use that the monadic theory of the full binary tree is decidable.*

Exercise 5 *Let K be a Kripke structure, b a node of K and φ be a formula in $TL(Until)$. Show that if K has an ω-path from b which satisfies φ then K has a quasi-periodic ω-path from b with the period bounded by $l \times u \times 2^{|\varphi|}$, which satisfies φ, where l is the length of the longest simple path from b, u is the number of occurrences of $Until$ in φ, and $|\varphi|$ is twice the number of subformula of φ.*
Note; *$a_1 a_2 \ldots a_i \ldots$ is quasi-periodic with a period l if there is N such that $a_n = a_{n+l}$ for all $n > N$.*

GOOD LUCK