1. Show that the problem $Gap - 3SAT[\frac{11}{16} + \varepsilon, 3/4]$ is $NP - Hard$. Note that we consider here the non-exact version, that is the input formula might contain clauses of size smaller than 3.

2. Consider the following variation of CSG_E, called *projection game*. For every edge uv and $\alpha \in \Sigma$, if u is assigned to α there is exactly one assignment to v such that the edge uv is satisfied and if v is assigned to α there is exactly one assignment to u such that the edge uv is satisfied. Denote by $Gap - kPG[a, b]$ the problem of deciding whether given an instance of a projection game, is it possible to satisfy at least b-fraction of the edges or every assignment satisfies at most a-fraction of the edges.

   Prove or disprove, assuming $P \neq NP$: For every $\varepsilon > 0$, $Gap - 7PG[\varepsilon, 1 - \varepsilon]$ is $NP - Hard$.

3. Suppose that you are given a randomized algorithm A that decides L in the following way:

   - If $x \in L$ then A accepts x with probability at least 0.2
   - If $x \notin L$ then A accepts x with probability at most 0.1

   where the probability is over the random coins of A. Provide an algorithm B that given x, if x is in L then B accepts with probability at least 0.9 and otherwise rejects with probability at least 0.9.

4. In this problem we deal with the *conditional expectations* method, which is used to transform randomized algorithms to deterministic ones. Let G be a simple graph on n vertices $x_1, x_2, \ldots, x_n$ and m edges.

   (a) Reprove that G contains a cut of size $m/2$. In other words, one can color the vertices in two colors, red and blue, such that at least half of the edges are between red and blue vertices.

   (b) For a fixed coloring for the vertices $x_1, \ldots, x_k$ let S_k be the expected number of edges that connect a red vertex and a blue vertex, where each vertex from $\{x_{k+1}, \ldots, x_n\}$ is red with probability $1/2$ and blue with probability $1/2$ independently. Show that $S_0 \geq m/2$ and that there is a choice of a color for x_{k+1} such that $S_{k+1} \geq S_k$.

   (c) Show that for a given coloring for the vertices $x_1, \ldots, x_k$, S_k can be calculated in polynomial running time.

   (d) Use the previous items to show a 2-approximation deterministic polynomial time algorithm for the max cut problem.

   (e) Use a similar method to provide a deterministic $16/15$-approximation algorithm for the Max-E4SAT problem.