1. Recall that for a graph $G$, $R(G)$ is the limit, as $k$ tends to infinity, of $[\chi(G^k)]^{1/k}$. What is the value of $R(C_5)$, where $C_5$ is a cycle of length 5?

2. Let $G_n = (V,E)$ be the graph of the $n$-cube, that is, $V = Z_2^n$ and two vertices are adjacent iff they differ in exactly one coordinate. What is the Shannon capacity $c(G_n)$ of $G_n$? What is the Witsenhausen rate $R(G_n)$ of $G_n$?

3. An automorphism of a graph $G = (V,E)$ is a one-to-one function from $V$ to $V$ that maps edges to edges. $G$ is called vertex transitive if for any two distinct vertices $u,v$ of $G$ there is an automorphism of $G$ mapping $u$ to $v$. Show that for any vertex transitive graph $G = (V,E)$, $\chi^*(G) = \frac{|V|}{\alpha(G)}$, where $\chi^*(G)$ is the fractional chromatic number of $G$ and $\alpha(G)$ is the independence number of $G$.

4. Let $n > 10^6$ be a large square. Bob knows $n$ pairs $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ of binary vectors, each of length $n$, where for each $i$, the Hamming distance between $x_i$ and $y_i$ is at least $n - 0.5\sqrt{n}$. Alice knows one of the vectors of each pair, that is, she knows $z_1, z_2, \ldots, z_n$ where for each $i$, $z_i \in \{x_i, y_i\}$. Can Alice send Bob less than $10n$ bits that will enable him to identify all the $n$ vectors $z_i$ among his $2n$ vectors? (We assume that Bob and Alice can agree on a communication protocol ahead of time, and they both know in advance that the Hamming distance between each pair of vectors of Bob will be at least $n - 0.5\sqrt{n}$.)

5. Let $G = (V,E)$ be a graph of chromatic number $r$ on the set of vertices $V = \{1,2,\ldots,n\}$, and suppose that there is a proper vertex-coloring $f : V \mapsto \{1,2,\ldots,r\}$ of $G$ by $r$ colors so that for every two connected vertices $i, j$ with $i < j$, $f(i) < f(j)$. Let $L(G)$ be the graph whose vertices are all ordered pairs $(i,j)$, where $1 \leq i < j \leq n$ and $(i,j)$ is an edge of $G$. The vertices $(i,j)$ and $(i',j')$ of $L(G)$ are connected if and only if either $j = i'$ or $i = j'$.

(i) What is the chromatic number of $L(G)$?

(ii) Can $R(L(G))$ be bigger than 4?