# Knowledge Based Simplification of Termination Proofs

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## Extended Abstract

There are many known approaches, methods and techniques to tackle termination proofs of rewrite systems. One of the less frequently used (abstract) approaches is to systematically investigate which knowledge can be used *a priori* to simplify the given termination proof task (before actually trying to prove termination) without sacrificing, of course, the essential logical and operational properties of the rewrite system under consideration. To this end, there exist at least two different approaches:[1]

(1) Using known general / abstract / structural results that reduce termination of a given system to a simpler termination property of (subsystems of) the same system.

(2) Transformational techniques that reduce termination of a given system to termination of a transformed (usually simpler) one.

These two abstract approaches as well as combinations thereof are particularly useful in the sense that one may use them as optional preprocessing steps that simplify the given termination proof tasks, and potentially render certain problems tractable that would have been intractable with the used underlying termination proof approach (e.g., using some class of precedence-based reduction orderings like *rpos*, recursive path ordering with status).

In previous works we had already obtained various results of type (1), including in particular several criteria which guarantee the equivalence of (general) termination and weakened termination properties like innermost or weak termination, cf. e.g. [5, 7, 6]. The underlying abstract and structural results[2] of [5, 7]

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[1] Of course, the whole field of (non-)modularity analysis of termination properties of rewrite systems could explicitly be mentioned here, too. In fact, it is subsumed by (1).

[2] It seems that the first significant results of type (1) date back to **A. Church** [3] and **M.H.A. Newman** [12].

have turned out to be very powerful and useful, also in other contexts (cf. e.g. [10], [4], [1, 2], [11]).

Partially based on these results, more recent results along the line of (2) are presented in [9, 8]. These latter results provide the theoretical basis for sound (and automatic) preprocessing steps when proving termination (semi-completeness) of (orthogonal) non-overlapping rewrite systems and equational programs defined by such systems. Here, soundness means that the relevant logical and operational properties are preserved.

In this paper we investigate to what extent and how (some of) the above mentioned results of [5, 6, 7, 9] can be extended and generalized. In particular we focus on the following extensions / generalizations:

- Regarding the criteria for equivalence of termination and innermost (weak) termination in [5, 7, 6], we study the conditional case, i.e., consider 2-CTRSs and also the more general – and practically very important – case of 3-CTRSs (cf. e.g. [13],[14]) in which extra variables are allowed not only in conditions, but also in right-hand sides.

- Concerning the transformational approach of [9], where certain simplifications of right-hand sides and conditions were allowed, we show how to relax the conditions for simplifying right-hand sides, and moreover, instead of only 2-CTRSs we also consider 3-CTRSs.

In the talk we shall concentrate on the most significant progress made in the directions sketched above, as well as on substantial difficulties encountered.

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