Decidability of Termination of Certain One-Rule String Rewriting Systems Extended Abstract

Alfons GESER*

ICASE, NASA Langley Research Center, Hampton, VA.

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Despite its simple appearance, the termination problem of single string rewriting rules (given a string rewriting rule $\ell \to r$, do all $\ell \to r$ derivations terminate?) is neither known solvable nor known unsolvable. This is so even in the case of rules $\ell \to r$ where ℓ has no self-overlap. The first promising approach for decidability has been introduced by McNaughton.

First one determines the sets $A = \mathrm{OVL}(\ell, r)$ and $B = \mathrm{OVL}(r, \ell)$ of left and right overlaps, respectively, between ℓ and r. For any $\alpha \in A$ let ℓ_α and r_α be defined by $\alpha\ell_\alpha = \ell$ and $r = r_\alpha\alpha$. Symmetrically one defines strings ℓ_β and r_β for every $\beta \in B$. Now the absence of certain rewrite steps can be expressed by the absence of certain patterns of decompositions of ℓ and r. For instance $r = \beta\ell_{\alpha_1} \ldots \ell_{\alpha_n} w \ell_{\beta_1} \ldots \ell_{\beta_m} \alpha$ is a decomposition which indicates that r may be consumed, up to a trunk w, by $n + 1$ rewriting steps at the left and $m + 1$ rewriting steps to the right.

McNaughton and later Kobayashi et al. distinguish ascending levels of complication that may be encountered during rewriting:

$$\text{well-behaved} \subsetneq \text{gentle} \subsetneq \text{tame} \subsetneq \text{arbitrary one-rule SRS}$$

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In a well-behaved rewriting derivation, any occurrence of the right hand side r of the rule cannot be totally consumed by rewriting steps. In other words, the trunk w cannot occur in a rewrite redex. Consequently the domains of rewriting within a string are separated by "dead" strings. Using this property McNaughton solved the termination problem in the class of well-behaved rules (even for the overlapping case).

For the next level in the hierarchy, gentle rules, there is as yet no decision procedure. In a gentle derivation, r may indeed by consumed, but in a strongly restricted way. First a left portion and a right portion of r is consumed, which leaves a trunk w. Then a rewrite step applies to the occurrence $\ell = \alpha' w \beta'$ of ℓ where $\alpha' \in A$ and $\beta' \in B$. This slight change accounts for a substantial complication.

Kobayashi et al. introduce a translation from non-overlapping gentle rules into different string rewriting systems with the aim that (non-)termination of the latter is easier to prove.

We present a decidability result for non-overlapping, gentle rules with one left overlap and only regular right overlaps. Regular right overlaps means that the set of right overlaps is of the form $B = \{\beta(v\beta)^{j-1} \mid 1 \leq j \leq n\}$ for some nonempty string β, string v, and $n \geq 1$.

We prove that every such rule is either left or right barren, or is well-behaved (in which cases termination is known to be decidable), or it has one of five forms that can effectively be checked. For each of the five forms we apply Kobayashi et al.'s transformation. Each of the five resulting string rewriting systems is either proven terminating by a recursive path order or proven non-terminating by a loop argument.